MAYOR'S FUND FOR LONDON

HELPING YOUNG LONDONERS GROW

COUNT ON US

SECONDARY CHALLENGE

STUDENT WORKBOOK 2023

GET ENGAGED IN MATHS!

1. INTRODUCTION

Welcome to your **Count on Us Secondary Challenge** student workbook.

This book contains three parts:

1. An introduction to tell you about the activities and the tournament, together with a tracker to help you decide which maths topics you want to work on.
2. Details about tournament activities, with rules and how it works in a tournament setting, together with activities specifically for preparing for the tournament.
3. A large collection of additional activities to help you prepare for the tournament and to help you with your catch-up work and to add to your knowledge and skills in Key Stage 3 mathematics.

The **Count on Us Secondary Challenge** is a maths tournament involving over 4000 young people from across London, delivered by the Mayor's Fund for London in partnership with the Jack Petchey Foundation. We hope that by taking part, you will become more confident in maths, will develop your problem-solving skills and boost your maths skills too.

The **Secondary Challenge** is made up of four rounds in different areas of maths. We hope you will find them really good fun and you'll want to practise lots to get really good at them! In the summer term, your school will select a team of 5 people to represent you in the regional Heats.

This book will explain how all the activities work and give you everything you need to try them out, practise them and get really good at puzzling, problem-solving and fast-paced number skills. You will work on statistics and probability, geometry, algebra and different number skills, so everything that you do will help you with your ordinary maths lessons too.

The national curriculum in maths also expects you to develop three skills which are developed specifically in mathematics:

(i) Fluency: you can do maths quickly and accurately, mostly in your head (NO reaching for a calculator when you see simple numbers!)
(ii) Reasoning: you can see and describe how things work mathematically.
(iii) Problem-solving: you can find a mathematical way to solve problems.

The tournament has 4 rounds:

1	Statistics and Probability	This is in two parts (i) teams play the game of hedgehog, a strategic dice game needing thoughtful probabilistic thinking and (ii) Data-Chart-Analysis, a card matching activity looking at statistical data, illustrated with charts and analysed.
2	Geometry	You will play the game of GridLines Geometry, solving geometric problems in different categories using randomly dealt number cards.
3	Number	You will play the 24®Game. A card game requiring mental number manipulation in a variety of categories: whole numbers, integers and fractions and decimals.
4	Algebra	Your team will solve a series of algebra problems in a story-based context.

Use the tracker starting on the next page to choose activities. The activities are coded as R1, R2, R3 or R4 (those used in the tournament, e.g. R1 for round 1, R2 for round 2 etc. or CU (those for catch-up work only).

R2 activities are given more detail. The cards in the GridLines Geometry game are themselves coded by topic. M=Mensuration (area and perimeter), V=Volume (and similarity), P=Pythagoras and A=Angles. So, if you see R2V look for the V cards in the pack and so on.

The tracker covers all of the content of the Key Stage 3 National Curriculum. It has been simplified to include only everything you can work on independently.

Tracker

Number	R	CU
1. Place value for decimals, measures and integers of any size.		N1
2. Order positive and negative integers, decimals and fractions; on a number line and with symbols =, ≠, <, >, ≤, ≥		N2
3. Prime numbers, factors (or divisors), multiples, common factors, common multiples, HCF, LCM and prime factorisation.		N3
4. Use the four operations with formal written methods.		N4
5. Use the four operations with integers (+ve and -ve numbers).	R3	N5
6. Use the four operations with decimals.	R3	N6
7. Use the four operations with fractions.	R3	N7
8. Use priority of operations: brackets, powers, roots and reciprocals.	R3	BBC1
9. Integer powers & real roots (square, cube, 4, 5) as decimals and surds.	R3	N9
10. Standard form $A \times 10^n$ $1 \leq A < 10$, where n is +ve, -ve integer or 0.		BBC2
11. Convert decimals and fractions and percentages.	R3	N11
12. Fraction Operations.		N12
13. Percentage; definition, calculation, comparison, change, operation.		N13
14. Round numbers and measures to appropriate degrees of accuracy.		BBC3
15. Round to estimate & calculate possible resulting errors as $a < x \leq b$		BBC3
Algebra		
1. Algebraic notation: $ab, 3y, a^2, a^3, a^2b, \frac{a}{b}$, coefficients, brackets.	R4	A1
2. Substitute numerical values into formulae and expressions.	R4	BBC4
3. Use vocab: expressions, equations, inequalities, terms and factors.	R4	BBC4
4. Algebraic manipulations: collect like terms, multiply out brackets, take out common factors, expand products of binomials.	R4	A2
5. Rearrange formulae to change the subject.		BBC4
6. Use algebraic methods to solve linear equations in one variable.	R4	A6
7. Work with co-ordinates in all four quadrants.	R4	A7
8. Graphs of linear and quadratic functions of one variable.	R4	A8
9. Reduce linear equations to $y = mx + c$; gradients and intercepts.		A9
10. Use linear and quadratic graphs to estimate values and to find approximate solutions of simultaneous linear equations.		BBC5
11. Find approximate solutions to contextual problems from graphs, including piece-wise linear, exponential and reciprocal graphs.	R4	BBC5
12. Terms of arithmetic, geometric and other sequences; nth terms with term-to-term or a position-to-term rules.	R4	A12

Ratio, proportion and rates of change		
1. Standard units e.g. time, length, area, volume/capacity, mass and compound units e.g. speed, unit pricing and density to solve problems.		BBC6
2. Use scale factors, scale diagrams and maps.		BBC6
3. Use ratio notation, including reduction to simplest form.		R3
4. Divide quantities as part:part or part:whole ratio; express as a ratio.		R4
5. Direct and inverse proportion; graphical and algebraic.		BBC6
Geometry and measures		
1. Use perimeter, area and volume formulae for triangles, parallelograms, circles, trapezia, cuboids, other prisms.	R2M	G1
2. Ruler and compass constructions.		G3
3. Use conventional geometric terms and notations.	R2	
4. Criteria for congruence of triangles and similarity by enlargement.	R2V	
5. Properties of triangles, quadrilaterals, circles, and other plane figures.		G6
6. Translations, rotations and reflections applied to given figures.		G7
7. Angles at a point, angles at a point on a straight line, vertically opposite angles, alternate and corresponding angles.	R2A	G8
8. Angle sum in any polygon, and properties of regular polygons.	R2A	G9
9. Use angle facts, similarity/congruence, Pythagoras' Theorem to obtain simple proofs.		BBC7
10. Use Pythagoras' Theorem and trigonometric ratios.	R2P	G10
11. Solve problems in 3-D using the properties of solid shapes.		BBC8
Probability		
1. Record, describe and analyse probability experiments.	R1a	P1
2. Understand that the probabilities of all possible outcomes sum to 1.		P2
3. Calculate theoretical probabilities using sample spaces.		P3
Statistics		
1. One variable statistics: central tendency (mean, mode, median) and spread (range, consideration of outliers).	R1b	S1
2. Statistical tables, charts and diagrams: frequency tables, bar charts, pie charts, pictograms and vertical line (or bar) charts.	R1b	S2
3. Two variables statistics using scatter graphs.	R1b	S3

2. TOURNAMENT ACTIVITIES

Professional mathematicians explore mathematics having no idea what the outcome might be. This needs them to be prepared to carry on even when they have no idea at all. They never give up. You need to develop this skill!

The English mathematician Andrew Wiles describes what this feels like in a BBC Horizon programme 'Fermat's Last Theorem', which can easily be found with an internet search. Just watch the first two minutes and you'll be hooked.

The first round consists of two independent activities. The first is to play the Game of Hedgehog, which needs you to decide on strategy using probability. In the second you will match sets of cards showing data sets, statistic charts and analysis.

ROUND 1a: The Game of Hedgehog

The Game of Hedgehog:

1. Two players or teams take turns.
2. In your turn: roll an ordinary die. If you roll 2 ,3, 4 or 5 then you score that amount. You can now choose to pass the turn to the other player or roll again. If you pass, you score the total you have made in this turn. If you roll again, you can add to your score if you roll 2, 3, 4 or 5. If you roll 1 then your turn over (but you can add the 1 to your score for this round). If you roll a 6, then you score zero for this round and your turn is over.
3. The first player (or team) to reach 30 points is the winner.

Example game:

Player 1 rolls 5 then 1 turns ends turn score 6	Player 1: 6	
Player 2 rolls 4 then 5 then 6 turn ends turn score 0	Player 1: 6	Player 2: 0
Player 1 rolls 4 then 3 then 1 turn ends turn score 8	Player 1: 14	Player 2: 0
Player 2 rolls 4 then 5 and passes turn score 9	Player 1: 14	Player 2: 9
Player 1 rolls 4 then 3 and passes turn ends turn score 7	Player 1: 19	Player 2: 9
Player 2 rolls 4 then 2 then 5 and passes turn score 11	Player 1: 19	Player 2: 20
Player 1 rolls 5 then 3 and passes turn score 8	Player 1: 27	Player 2: 20
Player 2 rolls 3 then 5 then 3 and wins(!) turn score 11	Player 1: 27	Player 2: 31

In team play (and in the tournament) a team of 5 plays as one player, playing in turn within their team. They roll the dice in turn. They can either roll or pass. They must not communicate with each other in any way.

You should play this game many times. Try to decide on a strategy. When should you carry on rolling and when should you pass? Think about the level of risk and the reward. In the tournament, some points are awarded for winning the round, but most points are awarded for your score. So, even if you lose, you will score well if you have a high score like Player 1 in the example game.

Team play is much harder than individual play. So, we will use this version of the game in the tournament. When you have fully learned this version, for individual play you can move on to the 2-dice game.

Two Dice Hedgehog:

1. Two players take turns.
2. In your turn: roll two ordinary die. If you roll 1, 2 ,3, 4 or 5 on both dice then you score the total amount rolled. You can now choose to pass the turn to the other player or roll again. If you pass, you score the total you have made in this turn. If you roll again, you can add to your score if you again roll 1, 2, 3, 4 or 5 on both dice. If you roll a 6 on either of the dice then you score zero for this round and your turn over. If you roll a double 6, then your total score zero is reset to zero and your turn is over.
3. The first player to reach 100 points is the winner.

Playing these games should help you get a sense for probability. You can find many more probability games here: https://nrich.maths.org/8494

ROUND 1b: Data-Chart-Analysis

In this round, you will receive an envelope containing 18 mixed up cards. They will show 6 different situations. For each situation there is a set of data, a chart illustrating that data and some summary Analysis showing, for example, the average and spread of the data. Your job will be to find the 6 sets each with data, chart and analysis.

Work through them but start with these activities from Nrich:

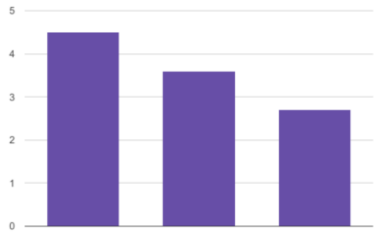
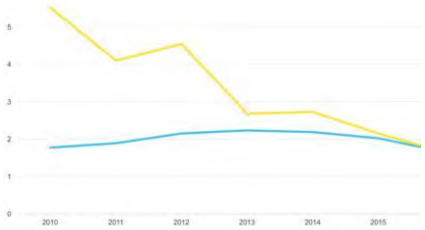
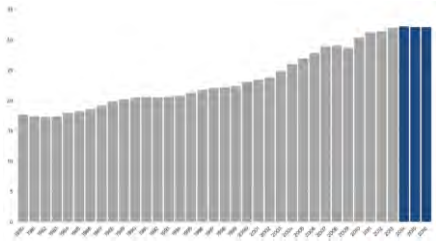
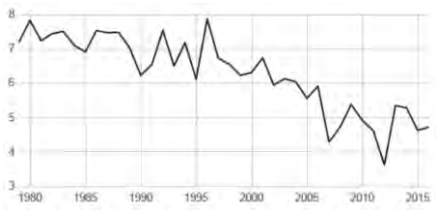
Data
What's the Weather Like? https://nrich.maths.org/whatstheweatherlike
Chart
Olympic records https://nrich.maths.org/records
Statistics
About Average https://nrich.maths.org/10995

In the tournament you will look at data from real world situations. In the news, in books on social media writers will make statements about information. They draw a conclusion (the Analysis) based on the data and often illustrate this with a chart. The relationship between these things is how statistics is used in practice.

You should search online for reports about things you are interested in, where charts and graphs have been used. Look at the relationship: Data-Chart-Analysis. This book from the European Union gives lots of examples to get you started: https://bit.ly/3EhgV8R.

Now try to match this sample set. Write an explanation to describe why you think the sets match. The answers are at the very end of this book. DO NOT look until you have thought hard to complete the matches!

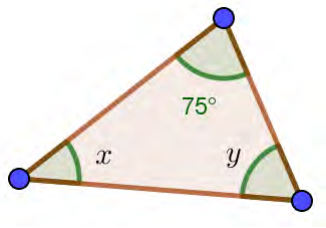
Data-Chart-Analysis Sample Card Set

Data Card D1	Chart Card C1	Analysis Card A1
Coastal tide gauges showing sea levels.		“Renewable energy is getting cheaper.”
Data Card D2	**Chart Card C2**	**Analysis Card A2**
The minimum amount of ice recorded in the Arctic each year.		“The Paris Agreement could make a huge difference to world temperatures.”
Data Card D3	**Chart Card C3**	**Analysis Card A3**
The amount of energy-related carbon dioxide emissions across the world.		“Arctic ice caps are melting.”
Data Card D4	**Chart Card C4**	**Analysis Card A4**
The cost of renewable energy in emerging-market economies.		“CO2 emissions are finally starting to level off.”
Data Card D5	**Chart Card C5**	**Analysis Card A5**
The breakdown of carbon emissions by country.		“Sea levels are rising.”
Data Card D6	**Chart Card C6**	**Analysis Card A6**
Forecast data compiled by Climate Analytics, ECOFYS, and others.		“China and the USA are responsible for the most carbon emissions.”

ROUND 2: Gridlines Geometry

In this round you will play a card game, designed specially for this tournament called *Gridlines Geometry*. It has lots of geometric situations in; (i) angle relationships, (ii) using Pythagoras' theorem, (iii) finding perimeters and areas, (iv) finding volumes and lengths in similar figures.

In the game you must find particular solutions to general situations, using some of the ten number cards you will have been dealt.
For example, here is a situation:



You know that $x + y = 180 - 75 = 105°$
So, you would try to make for two numbers that fit.
E.g. 50° and 55°, or 20° and 85°, or 37° and 68° etc.
If you cannot find numbers to fit, there will always be two more problems to work on.

To prepare for this round, you should get lots of practice solving standard geometry problems in the three areas.

1. Practise your geometric problem-solving.
    a. www.bbc.com/bitesize/guides/zrck7ty/revision/1
    b. www.bbc.com/bitesize/guides/z3g9q6f/revision/1
    c. www.bbc.com/bitesize/guides/z2mtyrd/revision/1
    d. www.bbc.com/bitesize/guides/zc9wxnb/revision/1

2. Get confident using variables in geometry
    a. Try this activity: nrich.maths.org/perimeterexpressions
    b. Work through this: www.ocr.org.uk/Images/222109-topic-check-in-6.01-algebraic-expressions.pdf
    c. Solve these: www.somerset.k12.ky.us/userfiles/103/Word%20Problems%20Perimeter%20and%20Age.pdf

3. Take the level 0 cards in the *Gridlines Geometry* game pack (look for a 0 in the blue circle on the card). Work with a partner. Take turns to find a set of numbers that fits the situation. Find as many sets as you can. Now find level 1 cards and repeat. Now, level 2. These are quite hard! Finally, find level 3 cards and repeat. These are very hard – you would be best working together.

4. Now play a game of *Gridlines Geometry*. Read the rules on the next two pages, so you are clear about how it works.

Gridlines Geometry Rules

Gridlines Geometry is a card game played with two 56 card decks consisting of:

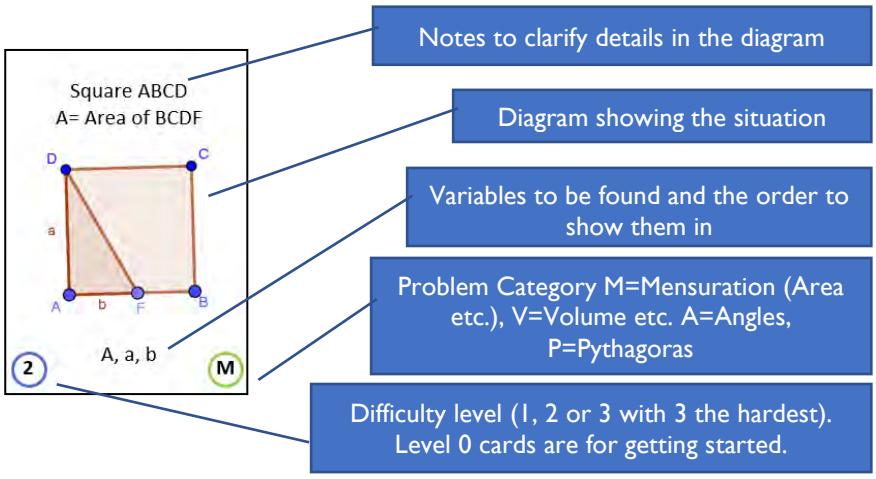
- 6 Rules cards (to remind you of the rules in play).
- 3 'I give up' cards (to use when you cannot find solutions to any problem).
- 44 Number cards.
- 9 Level 0 problems cards for getting started practice.
- 50 Geometric Problem cards.

The aim of the game is to find solutions to situations shown on **Problem Cards**, using numbers made from **Number Cards**. There are many possible solutions to each card. You must find numbers that can fit **all** the variables in the given situation.

Playing the Game Summary

1. Take the 3 'I Give Up' cards to use later.
2. Shuffle Number & Problem Card packs and place them separately face down.
3. Place top 10 Number Cards in 2 rows of 5 face up & 3 problem cards face up.
4. Solve Problem Cards using the Number Cards on the table. (Look at the *Example Solution* on the next page).
5. Take, keep and replace correctly solved Problem Cards. Return and replace used Number Cards.
6. Play an 'I Give Up' card at any time to replace any or all the number and/or Problem Cards.
7. Keep solving until the agreed time is up. Score 1 for each solved card.

The Problem Cards



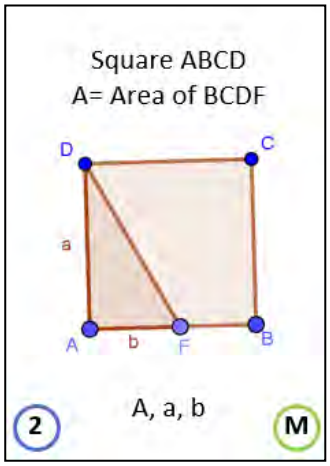
The Number Cards

Number Cards can be organised into groups or played individually. Put the cards on top of one another to make a new number. When placed on the table, ONLY the required number can be visible.

Examples

Use:	To Make	Or	Or
90 5	9 5	5 90	5 0
5 100 2	2100	1 2 5	210 5

Example Solution



I choose a=10 and b=4 because I can make 4 and 10 from the number cards.

- The area of the triangle AFD is $\frac{1}{2} \times 4 \times 10 = 20$
- The area of the square ABCD is $10^2 = 100$
- So, the area of BCDF is $100 - 20 = 80$

If I can make 80 from the number cards, then I can show my solution.

(If not, choose different values for a and b and try again).

I show A, a, b using the number cards for 80, 10, 4

You must use separate number cards for all the numbers.

A judge should check that the solution is correct.

Notes

1. In competitive play, you must explain, step-by-step to the judge how your variables fit the situation. You can show important calculations you have made in your notebook if this helps.
2. When you play one 'I give up!' card you can swap as many cards as you like. All of the problem **and** number cards or some number and/or some problem cards. Replace the cards taken to return to ten number cards and three problem cards face up on the table. Now resume play.
3. Experienced players may wish to count the total number of points from the cards solved. (These are shown in the blue circle on each card as 1, 2 or 3, where 3 is the hardest).

ROUND 3: The 24®Game. Whole Numbers, Fractions & Decimals, Algebra & Exponents

Ask an adult to do a calculation in their head and they'll run away! Everyone is scared of mental arithmetic. This round is designed to make sure YOU are not. All it needs is practice, practice, practice. (And a fun game to practise with …)

The 24® Game is a card game. Each card has 4 numbers on it. You have to combine the numbers using +, -, × or ÷ in any way you can to make an answer of 24. You MUST use all four numbers once and once only!

See if you can do it with these numbers:

4 5 8 4

Hints

- Try to find key number bonds: 6 x 4, 8 x 3, 16 + 8 …
- Try pairing the numbers up to make the parts you need.
- Try finding numbers to make 1 (to multiply and make no difference).
- Keep it all in your head!

Now try these:

1 5 2 6

2 3 6 2

2 6 2 8

1 5 3 9

2 5 4 6

Don't forget … we won't tell you the answers, so don't tell anyone else.

Use the next four pages to practise then use the 24®Game cards.

Different 24®Game Cards

There are different types of 24®Game cards with different number types. You will have some cards of the different types to practise, they are:

- Single Digits: 4 single digit numbers.
- Double Digits: one or more of the numbers will be a two digit number.
- Fractions and Decimals: one or more of the numbers will be expressed as a fraction or a decimal.
- Integers: one or more of the numbers will be negative.
- Algebra and Indices:
    - Algebra cards have one or more of the spaces replaced with an algebraic expression with variable x and/or y. You choose any number(s) for the variable(s) (not zero) and use the value of the expression(s) to complete the 24.
    - Indices cards have four index expressions (always $x^2, x^3, \sqrt{x}, \sqrt[3]{x}$). You choose one of these and apply one of the numbers on the card to make a new number, then use this and the others to complete the 24.

You should practise using each of the sets separately, then make sure you can still solve puzzles when the different types come up randomly. (Shuffle different types together to practise, but make sure to separate the packs to put them away!)

TORTURE SQUARES

Practise your fractions calculations with these ***torture squares***.

Complete each square using the operation shown: +, -, × or ÷

Do them at different times.

Allow 10 minutes max to complete each square.

+	3	$\frac{1}{2}$		4	$\frac{3}{4}$
$\frac{1}{3}$					
		$\frac{4}{5}$			
					$1\frac{1}{4}$
5			$6\frac{1}{2}$		
$\frac{2}{3}$					

×	8	0.1	2	$\frac{1}{4}$	
$\frac{1}{2}$					
0.3					
			0		
0.8					3.2
$\frac{1}{3}$					

first number					
-	4		$\frac{1}{4}$	7	0.3
1					
1.2					
			$2\frac{3}{4}$		
$\frac{1}{5}$		2.2			
2					

first number					
÷	6			$\frac{2}{3}$	0.5
2		0			
0.2					
$\frac{1}{4}$			4		
5					
$\frac{1}{3}$					

Make up more Torture Squares like these to test your friends.

FIND 24: THE BOARD GAME

1. You will need sets of counters of two different colours; one for each player.
2. Take turns to find 24 using any four numbers on the board.
   For example (On board 1 on the next page): Use $4, \frac{1}{2}, 10, 1.2$ make $4 \times \frac{1}{2} = 2$ and $10 \times 1.2 = 12$ then $2 \times 12 = 24$
   Use a timer to give a maximum of one minute per turn.
3. If you succeed, place 4 counters on the number you found.
4. If you fail, your opponent takes a turn.
5. Numbers cannot be used more than once.
6. When neither player can make 24 in two consecutive rounds, play ends and the winner is the player who has placed the most counters.

Alternative Rules:

1. Both players look for sets to make 24 at the same time.
2. If you find a set tap the table and play stops. Place your counters.
3. Score 1 if all 4 numbers are whole numbers, add one for each fraction or decimal you used.
4. Play until both players agree they cannot find any more sets OR agree a time limit in advance.
5. For an even harder game, the set of four numbers must be next to each other on the board (horizontally, vertically or diagonally).

***Exponents Version*:**

You **must** substitute *one* of your numbers (**but NOT the number 1**) into one of the following expressions: $x^2, x^3, \sqrt{x}, \sqrt[3]{x}$

***Algebra Version*:**

You must substitute one or two of your numbers into one of these expressions before making 24 (when an expression has been used it cannot be used again):

$x^2 + 1$	$x(x + 1)$	$x^2 + y$	$\frac{x^2}{y}$
$2x(1 - x)$	$\frac{x^2}{y^2}$	$x^2 + 2y^2$	$y(x^2 - 3)$
$x(x^2 + 2y)$	$\frac{x + 2y}{x}$	$x(3 - y^2)$	$\frac{3x - y}{2y}$

FIND 24 BOARD 1 (BEGINNER)

4	$\frac{1}{4}$	6	$\frac{1}{6}$	0.1	1
12	$\frac{1}{8}$	3	9	$\frac{1}{2}$	0.75
2	4	0.5	2	3	20
8	$\frac{2}{3}$	1	3	$\frac{1}{3}$	0.5
$\frac{1}{4}$	9	$\frac{3}{4}$	0.5	2	1
6	$\frac{1}{2}$	4	10	0.25	6

FIND 24 BOARD 2 (INTERMEDIATE)

6	$\frac{1}{4}$	3	1	$\frac{1}{3}$	8
$\frac{1}{4}$	-2	0.5	$\frac{1}{2}$	9	$\frac{1}{2}$
-12	$\frac{1}{8}$	0.1	4	0.5	20
3	0.2	$\frac{2}{3}$	-3	$\frac{1}{6}$	10
$\frac{1}{12}$	-9	-4	$\frac{5}{6}$	2	$\frac{3}{4}$
1.5	$\frac{2}{3}$	4	6	0.5	-6

FIND 24 BOARD 3 (EXPERT)

-7	$-\frac{1}{2}$	3	0.8	$\frac{3}{5}$	1.5
$\frac{1}{3}$	-2	0.5	$\frac{1}{4}$	-8	$\frac{1}{12}$
$2\frac{1}{4}$	$\frac{2}{3}$	-1.2	$1\frac{1}{3}$	2.5	11
4	-0.4	$\frac{3}{8}$	2	$\frac{1}{6}$	$-1\frac{3}{4}$
$\frac{1}{2}$	9	1.3	$-\frac{2}{3}$	1.4	$\frac{5}{12}$
0.3	$\frac{5}{6}$	-3	$1\frac{1}{2}$	0.8	5

ROUND 4: The Algebra Problem-Solving Challenge.

Algebra is at the heart of all mathematics. It is the language that mathematicians use. You must speak it fluently! Also, you live in London, one of the world's greatest cities and you need to know it well. In this round, you need to use your fast-paced skill in algebra to decode messages to solve a problem about the city.

1. Read about your great city online to get an idea of the things that people think are important, but don't worry, all of the information you need will be given to you in the round.
   - Go for a walk in London looking at maths: www.themathszone.com/?p=641
   - Read about London at: en.wikipedia.org/wiki/London
2. You will need to practise your algebra. This list shows all the algebra problems you will need to know. Use it to decide what to practise.
   - Algebraic notation: $ab, 3y, a^2, a^3, a^2b, \frac{a}{b}$, coefficients, brackets.
   - Substitute numerical values into formulae and expressions.
   - Use vocabulary: expressions, equations, inequalities, terms and factors.
   - Algebraic manipulations: collect like terms, multiply out brackets, take out common factors, expand products of binomials.
   - Use algebraic methods to solve linear equations in one variable.
   - Work with co-ordinates in all four quadrants.
   - Graphs of linear and quadratic functions of one variable.
   - Reduce linear equations to $y = mx + c$; gradients and intercepts.
   - Terms of arithmetic, geometric and other sequences; nth terms with term-to-term or a position-to-term rules.

Practise algebra by making and solving Tarsia puzzles.

- First you will need to download the free Tarsia software at: www.mmlsoft.com/index.php/products/tarsia
- Then download the Algebra set of puzzles (scroll down to find them) at: www.mrbartonmaths.com/jigsaw.htm
- When the software is installed, choose one of the puzzles. Look at the examples on the algebra page (later in this booklet) to guide your choice. Open the file. Make sure the 'output' tab is selected. Print out the sheets. Cut them out. Put them together to make a large hexagon so that edges match with question and answer. ONLY when you have finished click the 'solution' tab.
- We recommend you work with a partner to solve these puzzles.

Practise algebra by using GeoGebra.

- Go to https://www.geogebra.org/
- Click the START Calculator button. Try some things:
    - Type: solve(3x+17=5x+3) and press ENTER
    - Type: simplify(17x+5x-9x)
    - Type: factor(3x^2+6xy)
    - Type: y=3x+1 and press ENTER
    - Type: y=x^2+3x+1 and press ENTER
- In GeoGebra always use x, y and z for your variables.
- Try different equations and expressions. Experiment. Explore!

3. CATCH UP ACTIVITIES

The remainder of this book consists of workbook activities to help you practise your maths skills. You should use the tracker at the start of the book to help you find the page(s) for the skills you need to work on. We recommend that you print out a copy of the tracker so you can tick off activities as you complete them. The trackers shows the activity number N, A, R, G, S, P for Number, Algebra, Ratio, Geometry, Statistics, Probability. Some items have BBC references. In these cases, follow the links below to BBC bitesize activities.

All the activities are designed to explain what you should do. So, work through the activity on your own. The best way to use the pages is to print out what you need and write onto the sheet.

Answer sheets for checking when you are finished can be found on the Count on Us Challenge web site. Ask your teacher for a copy.

BBC References

No.	Tracker	Link
BBC1	N8	Operations: https://www.bbc.co.uk/bitesize/topics/znmtsbk
BBC2	N10	Standard Form: https://www.bbc.co.uk/bitesize/topics/z2mf9j6
BBC3	N14 & N15	Rounding and Estimating: https://www.bbc.co.uk/bitesize/topics/zmdqxnb
BBC4	A1 & A2 & A5	Expressions and Formulae: https://www.bbc.co.uk/bitesize/topics/z9yb4wx
BBC5	A10 & A11	Graphs: https://www.bbc.co.uk/bitesize/topics/zdbc87h
BBC6	R1 & R2 & R5	Ratio and Proportion: https://www.bbc.co.uk/bitesize/topics/zxw76sg
BBC7	G9	Angles: https://www.bbc.co.uk/bitesize/topics/zdr9wmn
BBC8	G11	2D & 3D Shapes: https://www.bbc.co.uk/bitesize/guides/zj76fg8/revision/3

N1 Place Value

Multiplying by 10, 100, and 1000

	ten thousands	thousands	hundreds	tens	units	point	tenths	hundredths
				8	2	.	3	6
x10			8	2	3	.	6	
x100		8	2	3	6	.		
x1000	8	2	3	6	0	.		

Dividing by 10, 100, 1000

	ten thousands	thousands	hundreds	tens	units	point	tenths	hundredths	thousandths
	7	8	6	4	3	.			
÷10		7	8	6	4	.	3		
÷100			7	8	6	.	4	3	
÷1000				7	8	.	6	4	3

Exercise 1

					5	.	6	4
x10						.		
x100						.		
x1000						.		

	6	3	4	.	2			
÷10				.				
÷100				.				
÷1000			0	.				

4.5 x 10 = _____ 34 ÷ 10 = _____ 0.12 x 10 = _____ 5.6 ÷ 10 = _____

0.74 x 100 = _____ 73 ÷ 100 = _____ 4.5 x 1000 = _____ 913 ÷ 100 = _____

442.7 x 1000 = _____ 4518 ÷ 1000 = _____ 0.55 x 100 = _____ 13 ÷ 1000 = _____

Multiplying					Dividing					
6	x	8	=	48	56	÷	8	=	7	
60	x	8	=	480	560	÷	8	=	70	
60	x	80	=	4800	560	÷	80	=	7	(560 ÷ 80 = 7)
600	x	8	=	4800	5600	÷	8	=	700	
600	x	80	=	48000	5600	÷	80	=	70	(5600 ÷ 80 = 70)
6000	x	800	=	4800000	5600	÷	800	=	7	(5600 ÷ 800 = 7)

Exercise 2

450 ÷ 9 = ____ 3 x 80 = ____ 640 ÷ 80 = ____ 4000 x 60 = ____

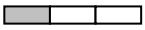
7200 ÷ 900 = ____ 300 x 60 = ____ 4200 ÷ 6 = ____ 80 x 40000 = ____

4900 ÷ 70 = ____ 500 x 6 = ____ 40000 ÷ 800 = ____ 700 x 80000 = ____

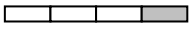
N2 Fractions

Fractions:

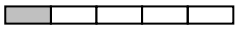
$\frac{1}{2}$ a half (one half)



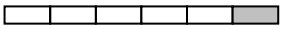
$\frac{1}{3}$ a third



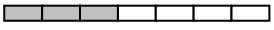
$\frac{1}{4}$ a quarter



$\frac{1}{5}$ a fifth



$\frac{1}{6}$ a sixth



$\frac{3}{7}$ three sevenths

$\frac{9}{10}$ nine tenths

$\frac{17}{20}$ seventeen twentieths

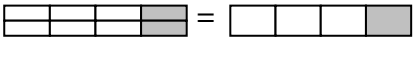
Exercise 1 Finish the table:

Fractions

	$\frac{3}{4}$	three quarters
		
		four ninths
	$\frac{3}{8}$	
	$\frac{11}{12}$	
		two thirds
		

Equivalent Fractions

These fractions are **equivalent**



$\frac{2}{8} = \frac{1}{4}$ two eighths is **equivalent** to one quarter

Exercise 2

$\frac{6}{8} = \frac{\quad}{4}$	$\frac{5}{8} = \frac{\quad}{16}$	$\frac{3}{8} = \frac{\quad}{40}$	$\frac{7}{10} = \frac{\quad}{40}$	$\frac{\quad}{7} = \frac{24}{28}$	$\frac{3}{20} = \frac{\quad}{180}$
$\frac{6}{\quad} = \frac{12}{14}$	$\frac{3}{8} = \frac{30}{\quad}$	$\frac{6}{9} = \frac{2}{\quad}$	$\frac{12}{\quad} = \frac{3}{10}$	$\frac{15}{\quad} = \frac{3}{7}$	$\frac{24}{\quad} = \frac{8}{10}$

Simplifying Fractions

Making the numbers smaller is called **simplifying**

$\frac{10}{20}=\frac{1}{2}$ $\frac{8}{12}=\frac{4}{6}$ $\frac{8}{12}=\frac{2}{3}$ $\frac{16}{20}=\frac{4}{5}$ $\frac{30}{80}=\frac{15}{40}$ $\frac{30}{80}=\frac{3}{8}$

$\frac{18}{45}\overset{\div 9}{=}\frac{2}{5}$ $\frac{96}{240}\overset{\div 2}{=}\frac{48}{120}\overset{\div 12}{=}\frac{4}{10}\overset{\div 2}{=}\frac{2}{5}$

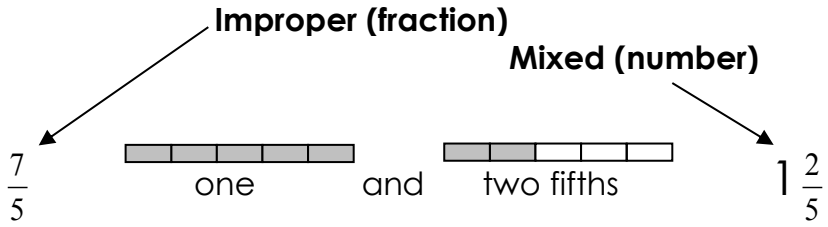
Exercise 3

Simplify:

$\frac{40}{50}=\frac{\quad}{\quad}$ $\frac{8}{16}=\frac{\quad}{\quad}$ $\frac{12}{16}=\frac{\quad}{\quad}$ $\frac{12}{20}=\frac{\quad}{\quad}$ $\frac{18}{24}=\frac{\quad}{\quad}$ $\frac{20}{48}=\frac{\quad}{\quad}$

$\frac{80}{96}=\frac{\quad}{\quad}$ $\frac{16}{120}=\frac{\quad}{\quad}$ $\frac{24}{144}=\frac{\quad}{\quad}$ $\frac{49}{140}=\frac{\quad}{\quad}$ $\frac{198}{360}=\frac{\quad}{\quad}$ $\frac{45}{225}=\frac{\quad}{\quad}$

Fractions greater than one



Exercise 4

Improper		Mixed
$\frac{11}{4}$	 two and three quarters	$2\frac{3}{4}$
$\frac{10}{3}$		
$\frac{11}{6}$		
		$2\frac{1}{5}$
		$3\frac{1}{7}$
		$4\frac{2}{3}$

N3 Factors, Multiples and Primes

Exercise 1

Multiples

These are **multiples** of two: 2, 4, 6, 8, ___ ___ ___

These are **multiples** of three: 3, ___ ___ ___ ___ ___ ___

These are **multiples** of ten: ___ ___ ___ ___ ___ ___ ___

Exercise 2

Factors

1 x **24** = **24** **2** x **12** = **24** **3** x **8** = **24** **4** x **6** = **24**

1, **2**, **3**, **4**, **6**, **8**, **12** and **24** are **factors** of **24**

1 x **18** = **18** ___ x ___ = **18** ___ x ___ = **18**

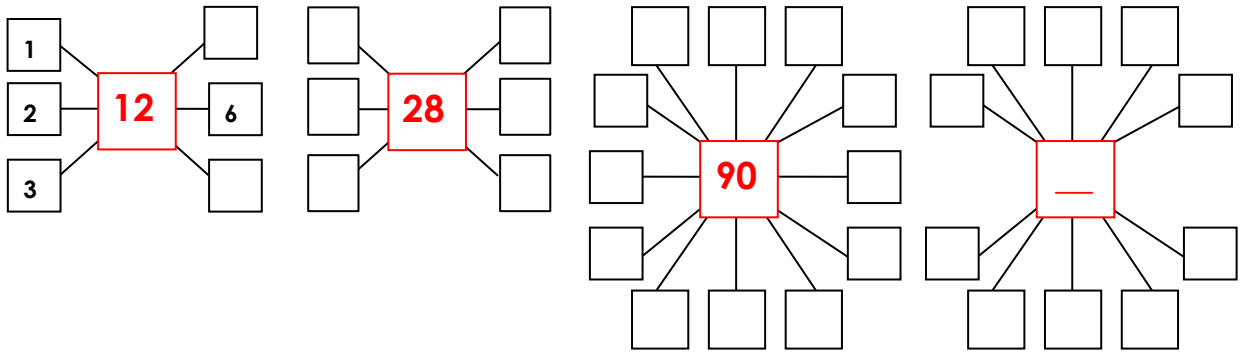
1, ___, ___, ___, ___ and **18** are **factors** of **18**

_____ are **factors** of **20**

_____ are **factors** of **16**

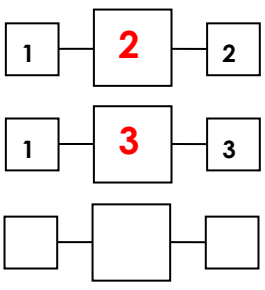
_____ are **factors** of **56**

Exercise 3



Exercise 4

Prime numbers have exactly two (2) different factors



2 is a prime number (1 x 2 = 2)

3 is a prime number (1 x 3 = 3)

___ is a prime number (___ x ___ = ___)

Exercise 5

These are the first 10 **prime numbers**: 2, 3, 5, __, __, __, __, __, __, 29

A prime number bigger than 50 (>50) is _____

A prime number bigger than 100 (>100) is _____

Exercise 6

Complete the table: write ✓ or ✖

	1	4	5	6	8	11	15	21	25	42	43	50
multiple of 2?	✖	✓	✖									
multiple of 3?	✖	✖	✖									
multiple of 5?	✖	✖	✓									
prime number?	✖	✖	✓									
factor of 50?	✓	✖	✓									

Exercise 7

Prime Factors

12 = 2 x 2 x 3 ← Prime Factors

= 2^2 x 3

36 = 3 x 3 x 2 x 2

= 3^2 x 2^2

__= __ x __ x __

= $__^2$ x __

__ = __ x __ x __ x __

= $__^2$ x __ x ___

Now find the **prime factors** of these numbers

30 _____ 100 _____ 72 _____ 105 _____ 33000 _____

Now find the **prime factors** of this number: 510510

The highest common factor (hcf)

The **factors** of 24 are 1, 2, 3, 4, 6, 8, 12, 24

The **factors** of 18 are 1, 2, 3, 6, 9, 18

The **common factors** of 18 and 24 are 1, 2, 3, **6**

The **highest common factor** of 18 and 24 is **6**

Exercise 8

The factors of 20 are ___, ___, ___, ___, ___, ____

The factors of 30 are ___, ___, ___, ___, ___, ____

The **common factors** of 20 and 30 are ___, ___, ___, ___,

The **highest common factor** of 20 and 30 is ____

The factors of 100 are _____

The factors 60 are _____

The **common factors** of 100 and 60 are _____

The **highest common factor** of 100 and 60 is _____

The highest common factor of 32 and 48 is _____

The highest common factor of 60 and 45 is _____

The lowest common multiple (lcm)

Multiples of 3 are 3, 6, 9, 12, 15, 18, 24, 27, 30, 33, 36, ...

Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...

Common multiples of 3 and 4 are **12**, 24, 36, ...

The **lowest common multiple** of 3 and 4 is **12**

Exercise 9

Multiples of 6 are ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, …

Multiples of 9 are ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, …

Common multiples of 6 and 9 are ___, ___, ___, ___, …

The **lowest common multiple** of 6 and 9 is ___

Multiples of 12 are ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, …

Multiples of 15 are ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, …

Common multiples of 12 and 15 are ___, ___, ___, ___, ___,

The **lowest common multiple** of 12 and 15 is ___

The lowest common multiple of 5 and 4 is ___

The lowest common multiple of 12 and 8 is ___

Exercise 10

Products

The **product** of 6 and 7 is 42 (6 x 7 = 42)

The **product** of 5 and 8 is ___ (__ x __ = __) The **product** of 9 and 7 is ___

Exercise 11

a	b	product	highest common factor (hcf)	lowest common multiple (lcm)
10	35			
15	12			
8	10			
12	16			
18	6			
7	11			

product ÷ _____ = _____

Simplify fractions

9 is the **highest common factor** of 18 and 45

$$\frac{18}{45} \overset{\div 9}{=} \frac{2}{5}$$

Exercise 12

Simplify

$\frac{32}{48} = \_\_$ $\frac{36}{54} = \_\_$ $\frac{16}{40} = \_\_$ $\frac{25}{30} = \_\_$ $\frac{63}{84} = \_\_$ $\frac{48}{72} = \_\_$

N4 Formal Written Methods

Adding

$$\begin{array}{r} 43 \\ +\ 35 \\ \hline 78 \end{array}$$

5 + 2 + 1 = 8

$$\begin{array}{r} {\scriptstyle 1} \\ 53 \\ +\ 29 \\ \hline 82 \end{array}$$

$$\begin{array}{r} \\ 453 \\ +\ 226 \\ \hline 679 \end{array}$$

3 + 9 = 12 so put the 10 here.

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 455 \\ +\ 769 \\ \hline 1224 \end{array}$$

5 + 9 = 14 so put the 10 here.

Exercise 1

$$\begin{array}{r} 12 \\ +\ 53 \\ \hline \end{array} \qquad \begin{array}{r} 38 \\ +\ 98 \\ \hline \end{array} \qquad \begin{array}{r} 553 \\ +\ 214 \\ \hline \end{array} \qquad \begin{array}{r} 789 \\ +\ 864 \\ \hline \end{array}$$

Subtracting

$$\begin{array}{r} 63 \\ -\ 32 \\ \hline 31 \end{array}$$

53 = 40 + 13

$$\begin{array}{r} {\scriptstyle 4\ 13} \\ \cancel{5}\ 3 \\ -\ 29 \\ \hline 24 \end{array}$$

13 – 9 = 4

$$\begin{array}{r} \\ 453 \\ -\ 221 \\ \hline 232 \end{array}$$

$$\begin{array}{r} {\scriptstyle 8\ 14\ 15} \\ \cancel{9}\ \cancel{5}.5 \\ -\ 76.9 \\ \hline 18.6 \end{array}$$

Exercise 2

$$\begin{array}{r} 66 \\ -\ 53 \\ \hline \end{array} \qquad \begin{array}{r} 78 \\ -\ 59 \\ \hline \end{array} \qquad \begin{array}{r} 553 \\ -\ 123 \\ \hline \end{array} \qquad \begin{array}{r} 91.1 \\ -\ 26.4 \\ \hline \end{array}$$

$$\begin{array}{r} 82 \\ -\ 53 \\ \hline \end{array} \qquad \begin{array}{r} 71 \\ -\ 28 \\ \hline \end{array} \qquad \begin{array}{r} 513 \\ -\ 427 \\ \hline \end{array} \qquad \begin{array}{r} 75.3 \\ -\ 19.8 \\ \hline \end{array}$$

4562 – 2167 = _____ 78.34 – 56.43 = _____ 639 – 376.43 = _____

Exercise 3

Multiplying

23 x 8

20 x 8 = 160
3 x 8 = 24
23 x 8 = 184

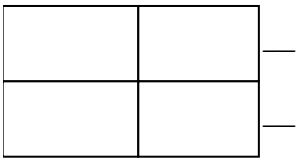
56 x 7 ___ x ___ = _____
___ x ___ = _____
56 x 7 = _____

47 x 6 ___ x ___ = _____
___ x ___ = _____
47 x 6 = _____

45 x 32 = 1200
150
80
+ 10
= 1440

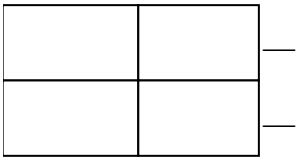
40	5	
40 x 30 =1200	5 x 30 =150	30
40 x 2 = 80	5 x 2 =10	2

63 x 74 = _____

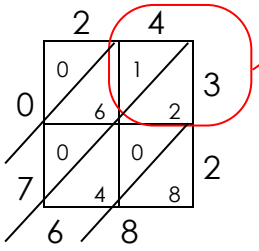


43 x 87 = _____

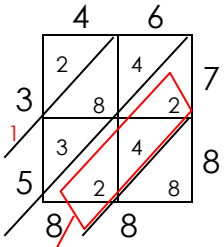
98 x 87 = _____



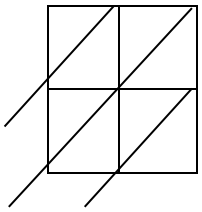
24 x 32 = 768



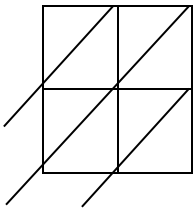
46 x 78 = 3588



37 x 43 = ____



47x 83 = _____

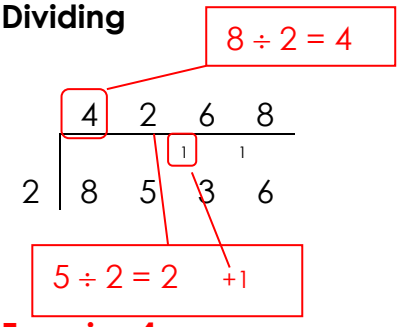


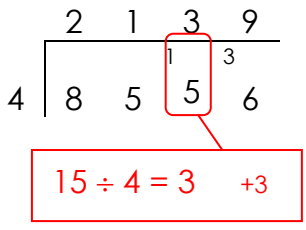
67 x 49 = _____

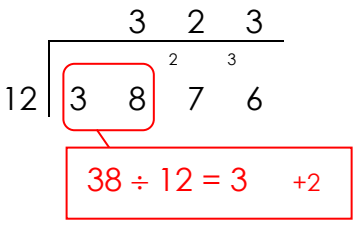
73 x 349 = _____

378 x 378 = _____

Dividing







Exercise 4

3 | 6 4 3 8

6 | 1 3 4 4

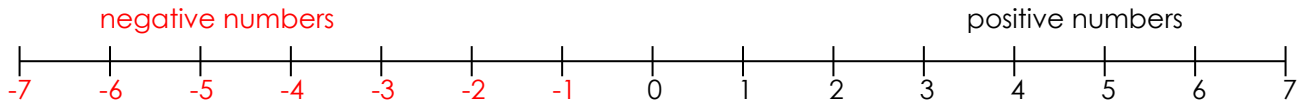
13 | 2 7 5 6

9765 ÷ 9 = _____

17010 ÷ 14 = _____

953226 ÷ 18 = _____

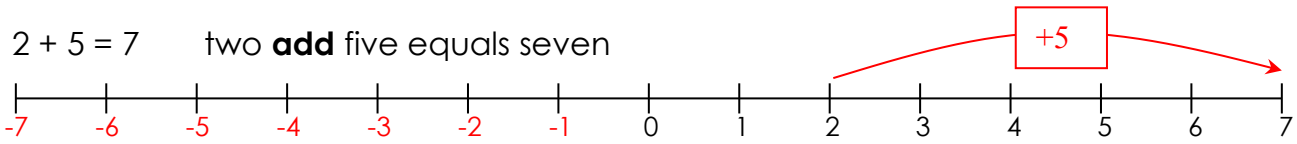
N5 Positive and Negative Numbers



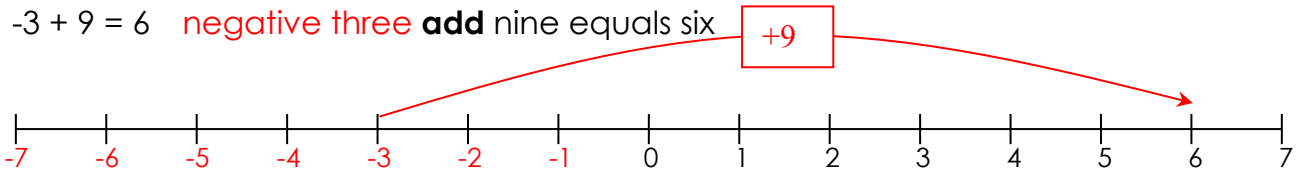
-7 negative seven -3 negative three -100 negative 100

Adding positive numbers

2 + 5 = 7 two **add** five equals seven

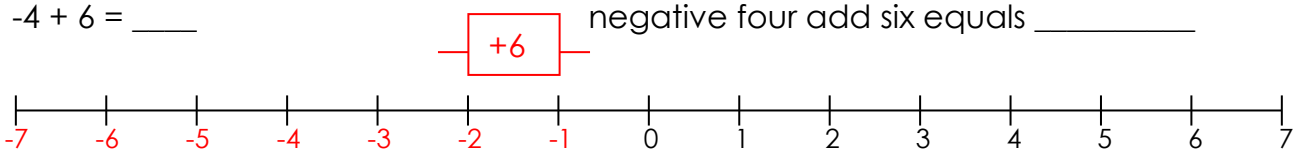


-3 + 9 = 6 negative three **add** nine equals six

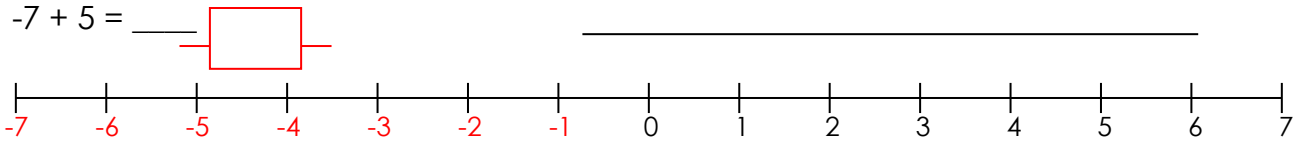


Exercise 1

-4 + 6 = ____ negative four add six equals _____



-7 + 5 = ____



-4 + 10 = ____ negative four add ten equals six

-1 + 6 = ____

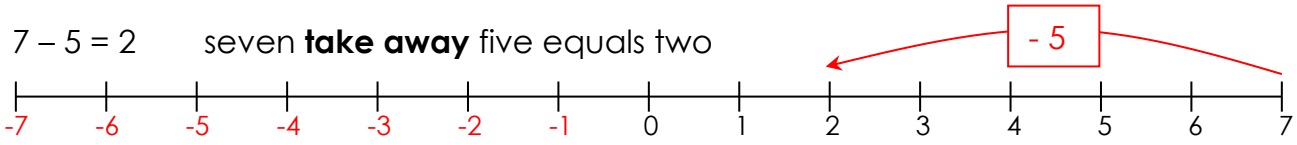
-7 + 7 = ____

-2 + 10 = ____

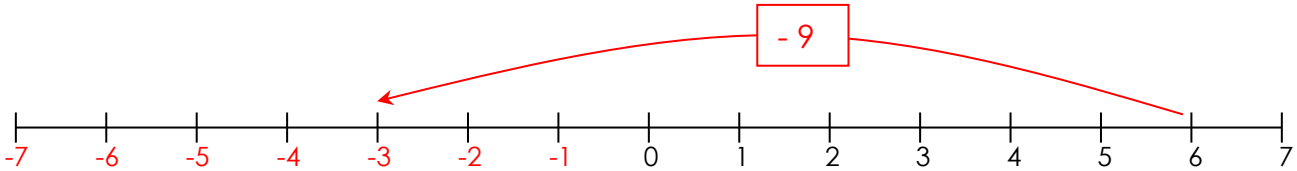
-5 + 11 = ___ -2 + 20 = ___ -42 + 100 = ___ -34 + 21 = ___ -46 + 11 = ___ -4 + 10 = ___

Subtracting positive numbers

7 – 5 = 2 seven **take away** five equals two

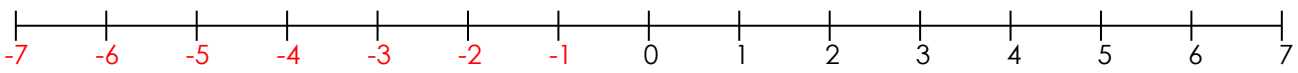


6 – 9 = -3 six **subtract** nine equals negative three



Exercise 2

3 – 8 = ____ three take away _____ equals _____



-1 – 4 = ____ negative one subtract _____ equals _____

4 – 10 = ____ four take away ten equals _____

2 – 9 = ____ _____

-3 – 4 = ____ _____

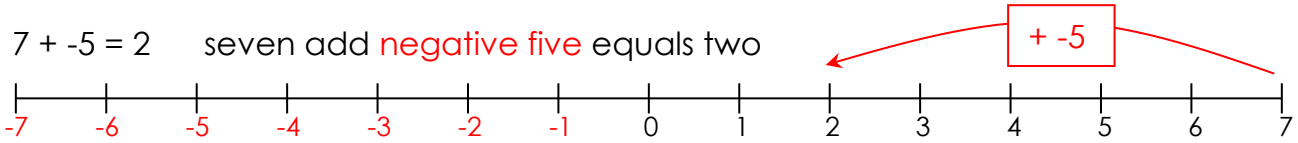
-4 – 2 = ____ _____

-4 – 12 = ____ _____

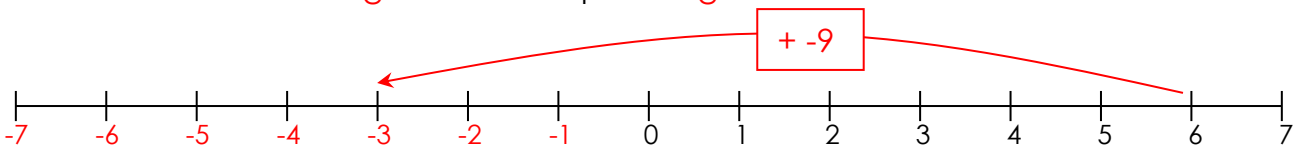
-5 – 26 = ____ 14 – 28 = ____ 29 – 42 = ____ -214 – 311 = ____ -56 – 100 = ____

Adding negative numbers

7 + -5 = 2 seven add negative five equals two

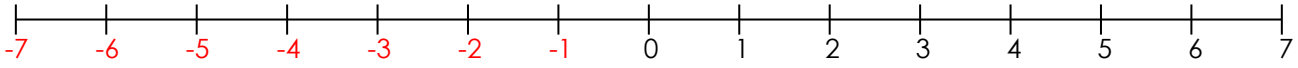


6 + -9 = -3 six add negative nine equals negative three



Exercise 3

3 + -8 = ____ three add _____ equals _____



-1 + -5 = ____ negative one add _____ equals _____

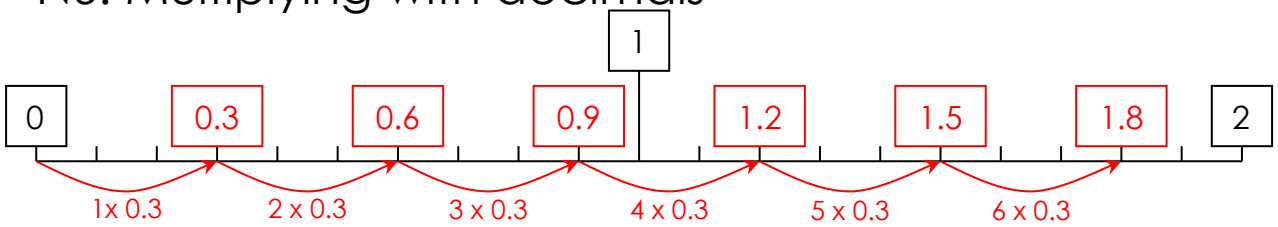
4 + -10 = ____ _____

2 + -9 = ____ _____

-3 + -4 = ____ -4 + -2 = ____ -5 + -46 = ____ 14 + -39 = ____

29 + -45 = ____ -245 + -422 = ____ -56 + -300 = ____

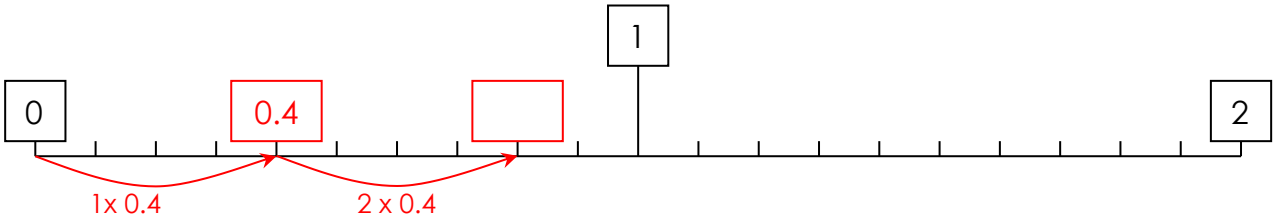
N6: Multiplying with decimals



1 x 0.3 = 0.3 3 x 0.3 = 0.9 5 x 0.3 = 1.5

2 x 0.3 = 0.6 4 x 0.3 = 1.2 6 x 0.3 = 1.8

Exercise 1



1 x 0.4 = ____ 3 x 0.4 = ____ 5 x 0.4 = ____

2 x 0.4 = ____ 4 x 0.4 = ____ 6 x 0.4 = ____

If 6 x 7 = 42 then 6 x 0.7 = 4.2 and 0.6 x 0.7 = 0.42

5 x 0.7 = ____ 0.5 x 0.7 = ____ 6 x 0.4 = ____ 0.6 x 0.4 = ____

9 x 0.3 = ____ 0.7 x 0.2 = ____ 8 x 0.3 = ____ 0.9 x 0.9 = ____

Dividing decimals

3 x 0.6 = 1.8 so 1.8 ÷ 3 = 0.6 18 ÷ 3 = 6 so 1.8 ÷ 3 = 0.6

Exercise 2

6 x 0.4 = ____ so ____ ÷ 6 = ____ 24 ÷ 6 = ____ so ____ ÷ 6 = ____

4.5 ÷ 9 = ____ 3.5 ÷ 7 = ____ 3.6 ÷ 4 = ____ 1.6 ÷ 2 = ____

6.3 ÷ 3 = ____ 7.2 ÷ 2 = ____ 8.4 ÷ 4 = ____ 9.9 ÷ 9 = ____

Dividing and multiplying by decimals

10 x 0.1 = 1 so 1 ÷ 0.1 = 10 30 x 0.2 = 6 so 6 ÷ 0.2 = 30

Exercise 3

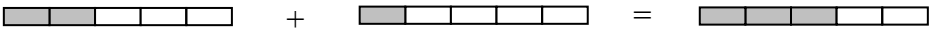
30 x 0.1 = ____ 3 ÷ 0.1 = ____ 40 x 0.2 = ____ 8 ÷ 0.2 = ____ 9 ÷ 0.3 = ____

5 ÷ 0.2 = ____ 10 ÷ 0.1 = ____ 14 ÷ 0.7 = ____ 15 ÷ 0.3 = ____ 80 ÷ 0.1 = ____

N7 Fraction Arithmetic

Add

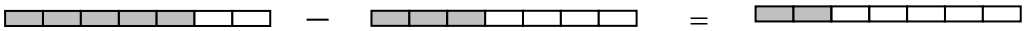
$\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$



two fifths add one fifth equals three fifths

Subtract

$\frac{5}{7}-\frac{3}{7}=\frac{2}{7}$



five sevenths subtract three sevenths equals two sevenths

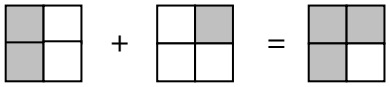
Exercise 1

$\frac{8}{9}-\frac{7}{9}=\_\_$	 eight ninths subtract _____ equals _____
$\frac{2}{6}+\frac{3}{6}=\_\_$	
$\frac{7}{10}-\frac{3}{10}=\_\_$	
$\frac{3}{8}+\frac{4}{8}=\_\_$	

BUT … $\frac{1}{2}+\frac{1}{4}=\frac{2}{6}$ ✗ … NO !

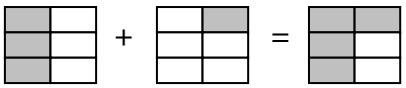
You must use **equivalent fractions** like this:

$\frac{1}{2}=\frac{2}{4}$ so $\frac{1}{2}+\frac{1}{4}=$ → $\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$

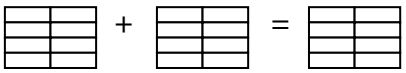


Exercise 2

$\frac{1}{2}=\frac{3}{6}$ so $\frac{1}{2}+\frac{1}{6}=$ → $\frac{3}{6}+\frac{1}{6}=\_\_$



$\frac{1}{2}=\frac{\_}{8}$ so $\frac{1}{2}+\frac{1}{8}=$ → $\frac{\_}{8}+\frac{1}{8}=\_\_$



$\frac{1}{2}=\_\_$ so $\frac{1}{2}+\frac{1}{10}=$ → $\_\_+\frac{1}{10}=\_\_$

$\frac{1}{3}=\frac{2}{6}$ so $\frac{1}{3}+\frac{1}{6}=$ → $\frac{2}{6}+\frac{1}{6}=\_\_$

$\frac{1}{3}=\_\_$ so $\frac{1}{3}+\frac{1}{9}=$ → $\_\_+\frac{1}{9}=\_\_$

$\frac{1}{4}=\_\_$ so $\frac{1}{4}+\frac{1}{8}=$ → $\_\_+\_\_=\_\_$

Adding fractions

These use **equivalent fractions** again:

$\frac{2}{3}+\frac{1}{4}=$ (x4, x3)

$\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$

$\frac{2}{5}+\frac{1}{3}=$ (x3, x5)

$\frac{6}{15}+\frac{5}{15}=\frac{11}{15}$

$\frac{2}{7}+\frac{2}{3}=$ (x3, x7)

$\frac{6}{21}+\frac{14}{21}=\frac{20}{21}$

Exercise 3

$\frac{3}{4}+\frac{1}{5}=$ $\quad \frac{\_\_}{20}+\frac{\_\_}{20}=\frac{\_\_}{20}$

$\frac{1}{7}+\frac{2}{5}=$ $\quad \frac{\_\_}{35}+\frac{\_\_}{35}=$

$\frac{2}{3}+\frac{3}{10}=$ $\quad \frac{\_\_}{30}+\frac{\_\_}{30}=$

$\frac{7}{10}+\frac{2}{11}=$ $\quad \frac{\_\_}{\_\_}+\frac{\_\_}{\_\_}=$

$\frac{4}{9}+\frac{3}{8}=$ $\quad \frac{\_\_}{\_\_}+\frac{\_\_}{\_\_}=$

Subtracting fractions

You **subtract** fractions in the same way

Exercise 4

$\frac{3}{4}-\frac{1}{5}=$ $\quad \frac{\_\_}{20}-\frac{\_\_}{20}=\frac{\_\_}{20}$

$\frac{6}{7}-\frac{2}{5}=$ $\quad \frac{\_\_}{35}-\frac{\_\_}{35}=$

$\frac{2}{3}-\frac{3}{10}=$ $\quad \frac{\_\_}{30}-\frac{\_\_}{30}=$

$\frac{7}{10}-\frac{2}{9}=$ $\quad \frac{\_\_}{\_\_}-\frac{\_\_}{\_\_}=$

$\frac{3}{8}-\frac{2}{7}=$ $\quad \frac{\_\_}{\_\_}-\frac{\_\_}{\_\_}=$

Multiplying fractions

Multiplying is easier: $\frac{6}{7}\times\frac{2}{5}=\frac{12}{35}$ $\quad \frac{5}{9}\times\frac{8}{10}=\frac{40}{90}=\frac{4}{9}$ (**simplify** the answer)

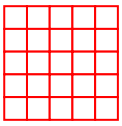
Exercise 5

$\frac{3}{4}\times\frac{5}{7}=\_\_$ $\quad \frac{2}{3}\times\frac{4}{5}=\_\_$ $\quad \frac{7}{8}\times\frac{3}{4}=\_\_$

$\frac{3}{4}\times\frac{2}{9}=\_\_$ $\quad \frac{1}{10}\times\frac{5}{7}=\_\_$ $\quad \frac{3}{4}\times\frac{8}{21}=\_\_$

N9 Powers and Roots

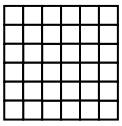
Squaring



5 x 5 = 25 $5^2 = 25$ five **squared** is twenty five

Exercise 1

___ x ___ = ___ $\_\_\_^2 = \_\_\_\_$ _____ **squared** is _____



___ x ___ = ___ _____ _____

ten squared is _____ $7^2 = \_\_\_\_$ eight squared is _____

$20^2 = \_\_\_\_$ fifteen squared is _____ $1.5^2 = \_\_\_\_$

Finding the square root

If $4^2 = 16$ then $\sqrt{16} = 4$

If four squared is sixteen then the **square root** of sixteen is four

Exercise 2

If $8^2 = \_\_\_\_$ then $\sqrt{64} = \_\_\_\_$

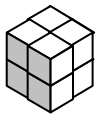
If eight squared is _____ then the **square root** of sixty four is _____

If $\_\_\_\_^2 = \_\_\_\_$ then $\sqrt{81} = \_\_\_\_$

If _____ then the **square root** of eighty one is _____

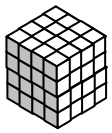
the **square root** of forty nine is _____ $\sqrt{400} = \_\_\_\_$ $\sqrt{121} = \_\_\_\_$ $\sqrt{¼} = \_\_\_\_$

Cubing



2 x 2 x 2 = 8 $2^3 = 8$ two **cubed** is eight

Exercise 3



___ x ___ x ___= ___ $\_\_\_^3 = \_\_\_\_$ _____ **cubed** is _____

___ x ___ x ___= ___ _____ _____

ten cubed is _____ $7^3 = \_\_\_\_$ fifteen cubed is _____ $20^3 = \_\_\_\_$

Exercise 4

If 5^3 = ___ then $\sqrt[3]{125}$ = ____

If five cubed is _____ then the **cube root** of one hundred and twenty five is _____

If ____^{3} = ____ then $\sqrt[3]{27}$ = ____

If _____ then the **cube root** of twenty seven is _____

the **cube root** of a thousand is _____ $\sqrt[3]{8000}$ = ____ $\sqrt[3]{343}$ = ____ $\sqrt[3]{\frac{1}{8}}$ = ____

Powers (or indices)

power or index

2^4 = $2 \times 2 \times 2 \times 2$ = 16

(1 2 3 4)

two **to the power** four is sixteen

Exercise 5

2^5 = __ x __ x __ x __ x __ = ____ two to the power _____ is _____

3^4 = _____ = ____ three_____ four is _____

1^6 = _____ = ____ _____

10^4 = _____ = ____ _____

5^4 = _____ = ____ _____

1^{56} = _____ = ____ _____

Squaring negative numbers

Remember negative x negative = positive (-3 x -4 = 12)

positive x negative = negative (5 x -2 = -10)

negative x positive = negative (-6 x 7 = -42)

Exercise 6

$(-4)^2$ = -4 x -4 = 16 minus four squared is sixteen

$(-10)^2$ = ___ x ___ = ____ _____ squared is _____

$(-1)^2$ = _____ _____ squared is _____

$(-8)^2$ = _____ _____ _____

Rules of Indices

Exercise 7

2^4	2 x 2 x 2 x 2	16
2^3		
2^2		
2^1		2
2^0		1
2^{-1}	$\frac{1}{2}$	$\frac{1}{2}$
2^{-2}	$\frac{1}{2x2}$	$\frac{1}{\ }$
2^{-3}	$\frac{1}{\ }$	
2^{-4}	$\frac{1}{\ }$	

÷ 2 ; x 2

10^4	10 x 10 x 10 x 10	10000
10^3		
10^2		
10^1		
10^0		1
10^{-1}		$\frac{1}{10}$
10^{-2}		
10^{-3}		
10^{-4}		

÷ 10 ; x 10

3^4		
3^3		
3^2		
3^1		
3^0		1
3^{-1}		
3^{-2}		
3^{-3}		
3^{-4}		

Power of **zero**:

$2^0 = 1$ $10^0 = \_\_\_$

$3^0 = \_\_\_$ $4^0 = \_\_\_$

$5^0 = \_\_\_$ $x^0 = \_\_\_$

Power of **one**:

$2^1 = 2$ $10^1 = \_\_\_$

$3^1 = \_\_\_$ $4^1 = \_\_\_$

$5^1 = \_\_\_$ $x^1 = \_\_\_$

Exercise 8

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $4^{-2} = \frac{1}{\_\_^2} = \frac{1}{\_\_\_}$ $5^{-3} = \frac{1}{\_\_^3} = \frac{1}{\_\_\_}$ $6^{-3} = \frac{1}{\_\_^3} = \frac{1}{\_\_\_}$ $x^{-2} = \frac{1}{\_\_\_^2}$

$2^{-6} = \frac{1}{\_\_} = \frac{1}{\_\_\_\_}$ $10^{-5} = \frac{1}{\_\_} = \frac{1}{\_\_\_}$ $9^{-2} =$ $4^{-3} =$ $x^{-6} =$

Multiplying with powers

$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3) = 3^6$ (4+2=6)

$7^6 \times 7^4 = (7 \times 7 \times 7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7 \times 7) = 7^{10}$ (6+4=10)

Exercise 9

You **add** the powers when you **multiply**. This is called **simplifying.**

$3^5 \times 3^3 =$ (___ x ___ x ___ x ___ x ___) x (___ x ___ x ___) = $3^{\square}$

$4^3 \times 4^5 =$ (_____) x (_____) = $4^{\square}$

$x^2 \times x^3 =$ _____ = $x^{\square}$

$x^3 \times x =$ (_____) x $x = x^{\square}$ (remember $x = x^1$)

Simplify $x^3 \times x^4 =$___ $x^7 \times x^7 =$___ $x^4 \times x^{-2} =$___ $x^8 \times x^{-7} =$___

$x^{-2} \times x^{-5} =$___ $x \times x =$___ $x^3 \times x =$___ $x^{-3} \times x =$___

But... $4^5 \times 3^4 \neq 12^9$ ... NO! You cannot simplify this.

Dividing with powers

If $3^4 \times 3^2 = 3^6$ then $3^6 \div 3^4 = 3^2$ (6 – 4 = 2)

If $5^4 \times 5^7 = 5^{11}$ then $5^{11} \div 5^7 = 5^4$ (11 – 7 = 4)

Exercise 10

You **subtract** the indices when you **divide**.

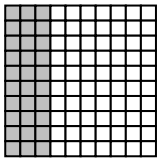
$3^5 \div 3^3 = \frac{(3 \times 3 \times 3 \times 3 \times 3)}{(3 \times 3 \times 3)} = 3^{\square}$ $4^3 \div 4^5 = \frac{(4 \times 4 \times 4)}{(4 \times 4 \times 4 \times 4 \times 4)} = 4^{\square}$

Remember $x = x^1$ and $x^0 = 1$

Simplify $x^9 \div x^4 =$___ $x^8 \div x^7 =$___ $x^4 \div x^7 =$___ $x^7 \div x^7 =$___

$x^5 \div x^2 =$___ $x \div x =$___ $x^3 \div x =$___ $x^{-3} \div x =$___

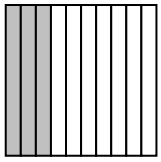
N11 Fractions, Decimals and Percentages



percentage | decimal | fraction | simplified

$30\% = 0.30 = \frac{30}{100} = \frac{3}{10}$

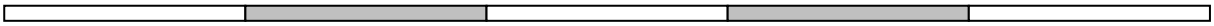
$\frac{30}{100}$ (hundredths) $\div 10$ $\frac{3}{10}$



hundredths

tenths

fifths



twentieths

Exercise 1

percentage	decimal	fraction /100	Simplified fraction
50%			
25%			
75%			
		$\frac{10}{100}$	
20%			
		$\frac{70}{100}$	
			$\frac{2}{5}$
5%			
65%			
	0.55		

$33\frac{1}{3}\% = 0.33333... = 0.\dot{3} = \frac{1}{3}$

percentage	decimal	fraction /100	Simplified fraction
	0.30		
		$\frac{80}{100}$	
			$\frac{9}{10}$
60%			
		$\frac{95}{100}$	
15%			
			$\frac{7}{20}$
	0.45		
		$\frac{85}{100}$	
1%			

$66\frac{2}{3}\% = 0.66666... = 0.\dot{6} = \frac{2}{3}$

Finding percentages

100%	£80
50%	£40
25%	£20
75%	£60
10%	£8
20%	£16
30%	£24
60%	£48
5%	£4
65%	£52

1. Find 75% of £80

Say:
seventy five percent of eighty pounds is sixty pounds
Write:
75% of £80 is £60

2. Find 65% of £80

Say:
sixty five percent of eighty pounds is fifty two pounds
Write:
65% of £80 is £52

Exercise 2

100%	£60
50%	
25%	
75%	
10%	
20%	
30%	
60%	
5%	
65%	

Percentages of £60

Say:
twenty five percent of sixty pounds is _____
Say:
_____
Say:
_____

Write:
30% of £60 is _____
Write:
_____
Write:
_____

100%	£96
50%	
25%	
75%	
10%	
20%	
33⅓%	
66⅔%	
5%	
2½%	
17½%	
1%	
6%	
27%	

100%	£270
	£27
	£54
	£90
	£162
	£81
	£13.50
	£202.50
	£180
	£67.50
	£2.70
50%	£135
	£6.75
	£47.25

£135 out of £270 is 50%

N12 Fraction Operations

Fraction of a number

$\frac{1}{2}\times 10 = 5$ $10 \div 2 = 5$ half of ten is five	$\frac{1}{3}\times 12 = 4$ $12 \div 3 = 4$ a third of twelve is four	$\frac{1}{4}\times 24 = 6$ $24 \div 4 = 6$ a quarter of twelve is three	$\frac{1}{8}\times 16 = 2$ $16 \div 8 = 2$ an eighth of sixteen is two

Exercise 1

$27 \div 3 =$ ___ $\frac{1}{3}\times 27 =$ ___	$100 \div 4 =$ ___ $\frac{1}{4}\times 100 =$ ___	$55 \div 5 =$ ___ $\frac{1}{5}\times 55 =$ ___	$18 \div 6 =$ ___ $\frac{1}{6}\times 18 =$ ___
a third of _____ is _____	a quarter of _____ is _____	a fifth of _____ is _____	a sixth of _____ is _____

$\frac{2}{3}\times 12 = ?$ 12 ÷ 3 = 4 so $\frac{1}{3}\times 12 = 4$ 4 x 2 = 8 so $\frac{2}{3}\times 12 = 8$

one third of twelve is **four** so **two** thirds of twelve equals **eight**

Exercise 2

$55 \div 5 =$ ___ $\frac{1}{5}\times 55 =$ ___ $\frac{4}{5}\times 55 =$ ___	$60 \div 10 =$ ___ $\frac{1}{10}\times 60 =$ ___ $\frac{3}{10}\times 60 =$ ___	$80 \div 8 =$ ___ $\frac{1}{8}\times 80 =$ ___ $\frac{5}{8}\times 80 =$ ___
$65 \div 5 =$ ___ $\frac{1}{5}\times 65 =$ ___ $\frac{2}{5}\times 65 =$ ___	$54 \div 9 =$ ___ $\frac{1}{9}\times 54 =$ ___ $\frac{8}{9}\times 54 =$ ___	$49 \div 7 =$ ___ $\frac{1}{7}\times 49 =$ ___ $\frac{6}{7}\times 49 =$ ___

$\frac{2}{3}\times 66 =$ _____ $\frac{2}{7}\times 42 =$ _____ $\frac{3}{8}\times 64 =$ _____

$\frac{7}{10}\times 12 =$ _____ $\frac{3}{4}\times 15 =$ _____ $\frac{7}{100}\times 500 =$ _____

N13 Percentage Operation

37% of 86

$37\% = \frac{37}{100}$

$\frac{1}{100} \times 86 = 0.86$

$37 \times 0.86 = \underline{\underline{31.82}}$

or

$37\% = \frac{37}{100}$

$\frac{37}{100} = 0.37$

$0.37 \times 86 = \underline{\underline{31.82}}$

Exercise 3

use a calculator

31 % of £75

__% = ___

$\frac{1}{\_\_\_} \times \_\_\_ = \_\_\_$

___ × _____ = _____

or

31% = _____ / _____

$\frac{31}{100} = \_\_\_\_\_$

_____ × _____ = _____

calculate

27% of £89 _____ 1% of £67.50 _____ 23% of £891 _____

Percentage increase

Increase £67 by 15%

calculate15% of £67 0.15 x 67 = 10.05

add the increase 67 + 10.05 = **£77.05**

Percentage decrease

Decrease £67 by 15%

calculate15% of £67 0.15 x 67 = 10.05

subtract the increase 67 - 10.05 = **£56.95**

Exercise 4

use a calculator

Increase £78 by 22%

calculate___% of £___ ___ x ___ = ___

add the increase ___ + _____ = _____

Decrease £12 by 43%

calculate___% of £___ ___ x ___ = _____

subtract the increase ___ - _____ = _____

Increase £952 by 67% _____

Decrease 67kg by 11% _____

Decrease £240 by 17% _____

Increase £32 by 27% _____

Decrease 54kg by 87% _____

Increase £22 by 17.5% _____

A1 Algebraic Notation

Exercise 1 Complete

a number add 1	$n + 1$		$1 + n$
two times a number	$2n$	$(n \times 2)$	
two numbers added together	$a + b$		$b + a$
two numbers multiplied together	st	$(s \times t)$	ts
one number take away another	$p - x$		
three times a number	_____		
a number add ten	_____		_____
three different numbers multiplied together	_____		_____
three different numbers added together	_____		_____
_____	$10n$		
half a number	_____		
_____	$c - 20$		
two numbers multiplied together then add 3	$ab + 3$	or	$3 + ab$
a number add 3 and then multiplied by 6	$6(n + 3)$		
_____	$10(d + 4)$		
_____	$10d - 7$		

Exercise 2

Substitution

a	b	x	$a + 7$	$5x$	$b - 9$	bx	$ab - x$	$a(b + x)$	____	____
4	7	10	11	50	-2	70	28 – 10 = 18	4 + 17 = 68	9	12
2	5	8							7	6
6	7	9								18
2	8	-6							10	

Exercise 3

Simplifying

$a + a = \underline{2a}$ $\quad 2a + 3a = \underline{5a}$ $\quad 3a + 6 + 4a - 10 = \underline{7a - 4}$ $\quad 2a + 3b + 6a - 8b = \underline{8a - 5b}$

1) $5b + 4b =$ _____ 2) $3a + a + 2b - b =$ _____ 3) $5b - 2a - b + 10a =$ _____

4) $3a + 3 + 4 =$ _____ 5) $3a - a - 2b - 6b =$ _____ 6) $3a + 3 - 4a =$ _____

7) $5b - 24b =$ _____ 8) $3a + 2b - a - 7b =$ _____ 9) $5 - 2a - 4 + 10 =$ _____

A4 Algebraic Manipulations

Brackets ⟶ ( )

5(2 + 4) = 5 x 6 = 30 (add then multiply)

5 x 2 + 4 = 10 + 4 = 14 (multiply then add)

Exercise 1

Expanding Brackets

a	b	c	$(b+c)$	$\mathbf{a(b+c)}$	ab	ac	$\mathbf{ab+ac}$
2	5	9	14	28	10	18	28
5	3	6					
7	7	8					
-5	6	8					
-3	2	6					

$a(b + c) = ab + ac$

$3(b + c) = 3a +$ ___

$3(5 + c) = 15 +$ ___

$4(a + 3) =$ _____

a	b	c	$(b-c)$	$\mathbf{a(b-c)}$	ab	ac	$\mathbf{ab-ac}$
2	10	3	7	14	20	6	14
5	6	3					
7	7	8					
-5	8	6					
-3	2	6					

$a(b - c) = ab - ac$

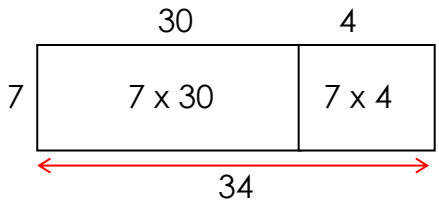
$5(b - c) = 5b -$ ___

$5(2 - c) = 10 -$ ___

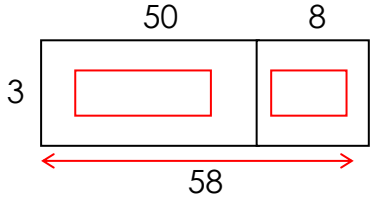
$7(a - 5) =$ _____

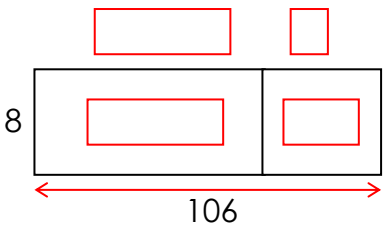
Exercise 2

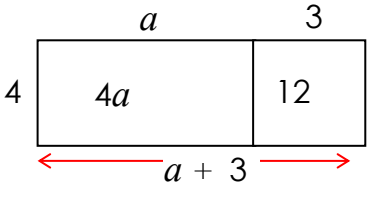
Brackets and Area



7 x 34 = 7 x 30 + 7 x 4 = 210 + 28 = 238





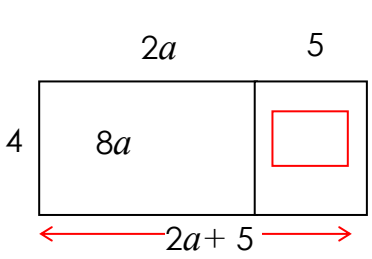


3 x 58 =

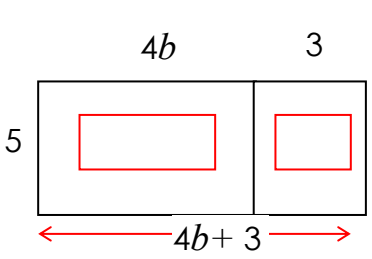
8 x 106 =

$4(a + 3) = 4a + 12$

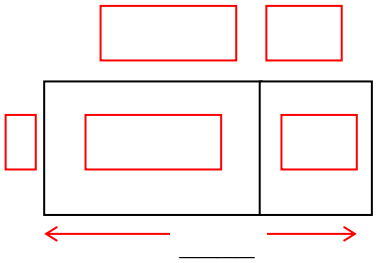
Exercise 3 Complete



$4(2a + 5) =$ _____



__(__ + __) =_____



7(2a + 9) =_____

$3(2a + 2) =$ $6a +$ _____ $4(3b - 5) =$ $12b -$ _____ $6(7x + 3) =$ $42x +$ _____

$4(5x - 8) =$ _____ $3(8a + 12) =$ _____ $4(2b - 15) =$ _____

$3(5x - 4y) =$ _____ $5(8a + b) =$ _____ $8(b - 5y) =$ _____

Factorising

$a(b + c)$ → **expanding brackets** → $ab + ac$ → **factorising** → $a(b + c)$

Exercise 4 Complete

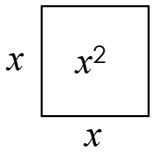
$2a + 2b = 2(a +$ ___$)$ $3a + 3b = 3($___ + ___$)$ $2a + 6 =$ ___$(a + 3)$

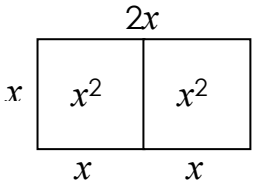
$12a + 20b = 4(3a +$ ___$)$ $21a + 12b = 3($___ + ___$)$ $2a - 2c =$ ___(___ – ___)

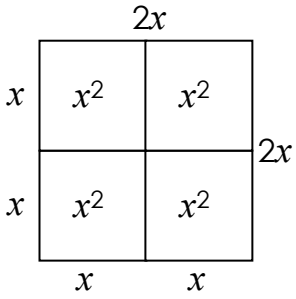
$10a + 20b =$ _____ $6a + 12b =$ _____ $4a - 2c =$ _____

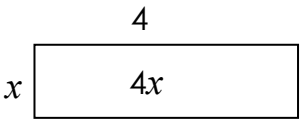
$8a + 12b =$ _____ $60a + 12b =$ _____ $14a - 21c =$ _____

Squares and quadratics



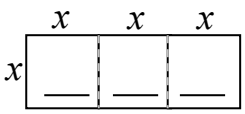


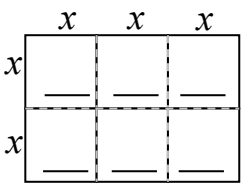


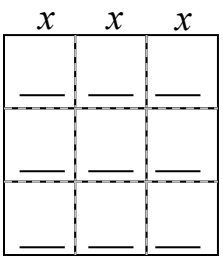


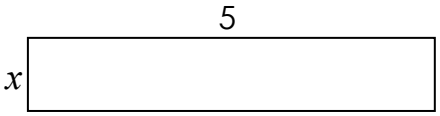
$x \times x = x^2$ $2x \times x = 2x^2$ $(2x)^2 = 2x \times 2x = 4x^2$ $4 \times x = 4x$

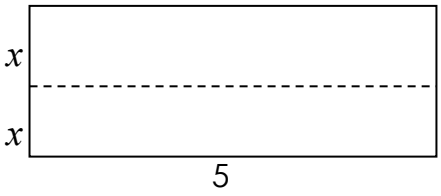
Exercise 5 Complete









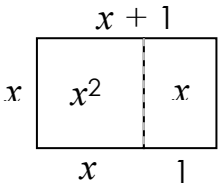


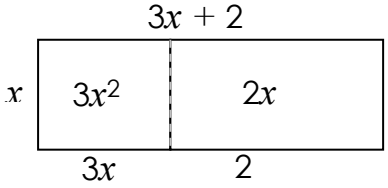
Simplify

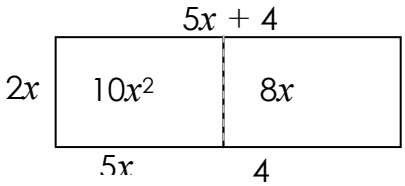
$3x \times x =$ ___ $3x \times 2x =$ ___ $(3x)^2 =$ __$x \times$ __$x =$ ___ $5 \times x =$ ___ $5 \times 2x =$ ___

$6x \times x =$ ___ $7x \times 4x =$ ___ $(4x)^2 =$ __$x \times$ __$x =$ ___ $8 \times x =$ ___ $8 \times 3x =$ ___

Expanding Brackets

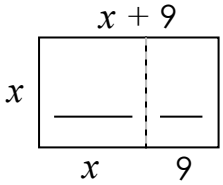


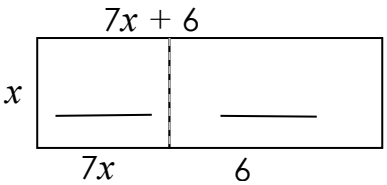


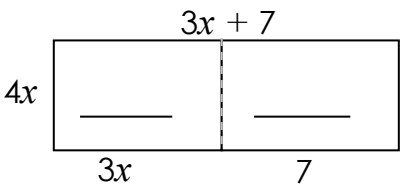


$x(x + 1) = x^2 + x$ $x(3x + 2) = 3x^2 + 2x$ $2x(5x + 4) = 10x^2 + 8x$

Exercise 6 Complete





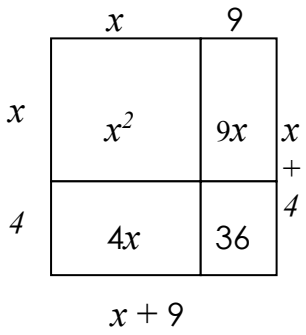


$x(x + 9) =$ _____ $x(7x + 6) =$ _____ $4x(3x + 7) =$ _____

$x(x + 3) =$ _____ $x(4x + 7) =$ _____ $3x(7x + 5) =$ _____

Expanding quadratics

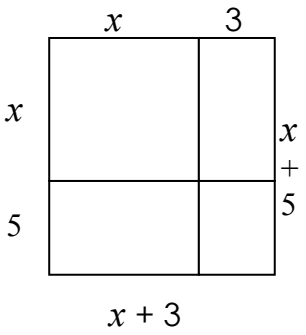
Exercise 7 Complete



$(x + 9)(x + 4)$

$= x^2 + 9x + 4x + 36$

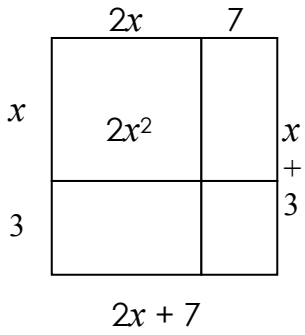
$= \underline{x^2 + 13x + 36}$



$(x + 3)(x + 5)$

$=$ _____

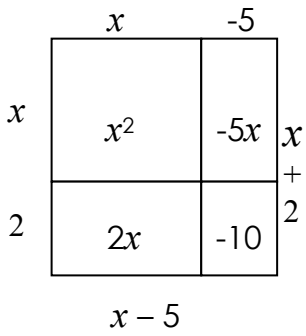
$=$ _____



$(2x + 7)(x + 3)$

$=$ _____

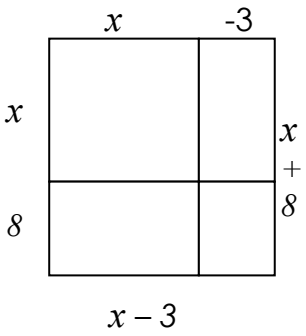
$=$ _____



$(x - 5)(x + 2)$

$= x^2 - 5x + 2x - 10$

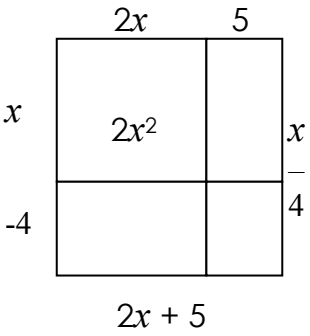
$= \underline{x^2 - 3x - 10}$



$(x - 3)(x + 8)$

$=$ _____

$=$ _____



$(2x + 5)(x - 4)$

$=$ _____

$=$ _____

Exercise 8

Expand these brackets

$(x - 3)(x + 4) =$ _____ $=$ _____

$(x + 7)(x + 4) =$ _____ $=$ _____

$(x + 7)(x + 6) =$ _____ $=$ _____

$(5x - 3)(x - 4) =$ _____ $=$ _____

$(x - 8)(3x - 4) =$ _____ $=$ _____

$(4x - 3)(x + 4) =$ _____ $=$ _____

Factorise these quadratics

$x^2 + 6x + 8 = (\_\_\_\_\_)(\_\_\_\_\_)$

$x^2 + 7x + 12 = (\_\_\_\_\_)(\_\_\_\_\_)$

$x^2 + 11x + 24 = (\_\_\_\_\_)(\_\_\_\_\_)$

$x^2 + 8x + 15 = (\_\_\_\_\_)(\_\_\_\_\_)$

A6 Solving Linear Equations

equation — solution

If $3x = 12$ then $x = 4$ This is **solving** an **equation**

Exercise 1 **Complete**

$x + 4 = 20$: -4, -4 → $x =$ ____

$x - 6 = 13$: +6, +6 → $x =$ ____

$5x = 35$: ÷5, ÷5 → $x =$ ____

$\frac{x}{3} = 4$: x 3, x 3 → $x =$ ____

$x + 17 = 20$: ____ → $x =$ ____

$x - 6.5 = 13$: ____ → $x =$ ____

$8x = 56$: ____ → $x =$ ____

$\frac{x}{4} = 21$: ____ → $x =$ ____

$3x + 5 = 26$ ← (*x* **multiplied by three** then **add five** equals twenty six)

-5, -5

$3x = 21$

÷3, ÷3

$x = 7$

$\frac{x}{4} + 7 = 17$ ← (*x* **divided by four** then **add 7** equals seventeen)

-7, -7

$\frac{x}{4} = 10$

x4, x4

$x = 40$

Exercise 2 **Complete**

$5x + 8 = 48$: -8, -8 → ____ $x =$ ____ → ____ → $x =$ ____

$4x - 5 = 79$: +5, +5 → ____ $x =$ ____ → ____ → $x =$ ____

$\frac{x}{3} + 14 = 30$: ____ → $\frac{x}{3} =$ ____ → ____ → $x =$ ____

$\frac{x}{4} - 27 = 67$: ____ → $\frac{x}{4} =$ ____ → ____ → $x =$ ____

$6x - 8 = 40$: ____ → ____ $x =$ ____ → ____ → $x =$ ____

$5x + 2 = 68$: ____ → ____ $x =$ ____ → ____ → $x =$ ____

$\frac{x}{10} - 20 = 10$: ____ → $\frac{x}{\ } =$ ____ → ____ → $x =$ ____

$\frac{x}{5} + 27 = 47$: ____ → $\frac{x}{\ } =$ ____ → ____ → $x =$ ____

Equations with brackets

$3(x + 5) = 63$ ÷3 ÷3

$x + 5 = 21$ -5 -5

$x = 16$

(x **add five** then **multiplied by three** equals sixty three)

$\frac{x+7}{4} = 6$ x4 x4

$x + 7 = 24$ -5 -5

$x = 17$

(x **add 7** then **divided by four** equals six)

Exercise 3 Complete

$6(x + 2) = 48$

$x + 2 = \_\_\_$

$x = \_\_\_$

$4(x - 10) = 84$

$x - 10 = \_\_\_$

$x = \_\_\_$

$\frac{x+5}{3} = 3$

$x + 5 = \_\_\_$

$x = \_\_\_$

$\frac{x-22}{4} = 9$

$x - 22 = \_\_\_$

$x = \_\_\_$

$7(x - 13) = 42$

$x = \_\_\_$

$5(x + 10) = 15$

$x = \_\_\_$

$\frac{x-13}{9} = 7$

$x = \_\_\_$

$\frac{x-42}{7} = 9$

$x = \_\_\_$

$\frac{3(x+7)}{4} = 60$ x4 x4

$3(x + 7) = 240$ ÷3 ÷3

$x + 7 = 80$ -7 -7

$x = 73$

Exercise 4 Solve these equations

1) $11x = 134$

2) $\frac{x}{4} + 7 = 17$

3) $12x - 6 = 30$

4) $3x - 8 = 100$

5) $\frac{x}{7} - 7 = 3$

6) $2x - 6 = 7$

7) $3(x - 8) = 12$

8) $\frac{x-7}{7} = -2$

9) $2(x - 6) = 102$

Exercise 5 Solve these equations

1) $\frac{5(x-8)}{4} = 10$

2) $\frac{7(x-7)}{3} = -21$

3) $\frac{2(x-6)}{10} = 102$

Solving Equations with x on Both Sides

$3x + 5 = x + 10$

$-x$ $-x$

$2x + 5 = 10$

-5 -5

$2x = 5$

$\div 2$ $\div 2$

$x = 2.5$

$3x + 18 = 7x - 10$

$-3x$ $-3x$

$18 = 4x - 10$

$+10$ $+10$

$28 = 4x$

$\div 4$ $\div 4$

$7 = x$

$x = 7$

$3x + 5 = 21 - x$

$+x$ $+x$

$4x + 5 = 21$

-5 -5

$4x = 16$

$\div 4$ $\div 4$

$x = 4$

$5 - 6x = 20 - x$

$+6x$ $+6x$

$5 = 20 + 5x$

-20 -20

$-15 = 5x$

$\div 5$ $\div 5$

$-3 = x$

$x = -3$

Exercise 6 Complete

$5x + 5 = x + 45$

$-x$ $-x$

$3x + 18 = 10x - 10$

$-3x$ $-3x$

$8x - 9 = 21 - 2x$

$+2x$ $+2x$

$4 - 9x = 20 - x$

$+9x$ $+9x$

Solve these equations

1) $7x + 5 = 4x + 50$

2) $8x + 18 = 10x - 11$

3) $3x - 9 = 21 - 2x$

4) $5 - 8x = 29 - 2x$

Exercise 7 **Solve these equations**

1) $8x - 3 = 5x - 45$ 2) $7x - 19 = 10x - 10$ 3) $x - 9 = 21 - 5x$ 4) $10 - 9x = 90 - x$

5) $3x + 5 = 2x + 5$ 6) $3x + 18 = 13x + 10$ 7) $8x - 1 = 1 - 2x$ 8) $4 - 9x = 3 - 7x$

Using brackets

$3(x + 5) = 2(x + 10)$

multiply out brackets

$3x + 15 = 2x + 20$

$x = 5$

$3(x - 1) = 2(3x - 6)$

multiply out brackets

$3x - 3 = 6x - 12$

$x = 3$

$6(x - 1) = 2(7 - 2x)$

multiply out brackets

$6x - 6 = 14 - 4x$

$x = 2$

Exercise 8 **Solve these equations**

1) $5(x + 5) = 2(x + 20)$

multiply out brackets

2) $4(x - 1) = 3(3x - 8)$

multiply out brackets

3) $5(x - 2) = 2(2 - x)$

multiply out brackets

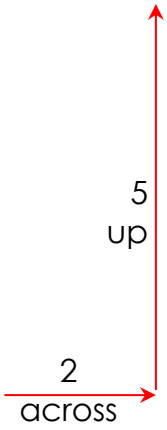
4) $5(2x - 3) = 3(x + 2)$

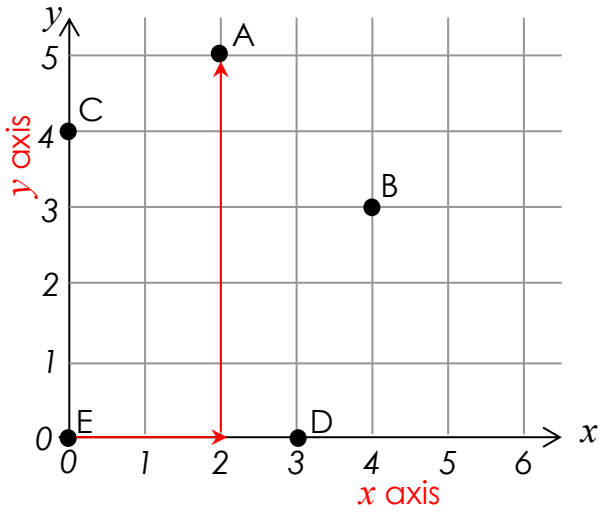
5) $7(x - 1) = 3(3x + 5)$

6) $4(x - 2) = 6(2 - x)$

A7 Graphs and Coordinates

The coordinates of A = (2 , 5)





Exercise 1

The coordinates of:

B = (___ , ___)

C = (___ , ___)

D = (___ , ___)

E = (___ , ___)

Exercise 2

Plot these **points** on the **grid**

P (2 ,1)

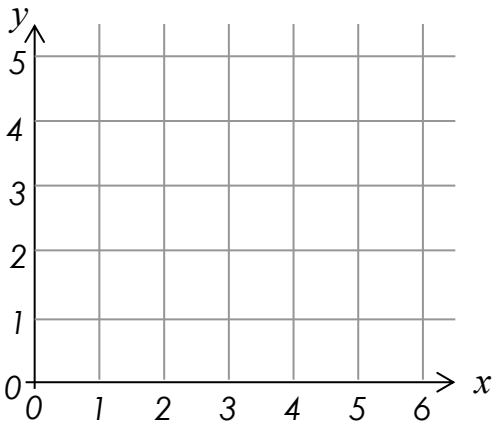
Q (1, 5)

R (0, 3)

S (6, 5)

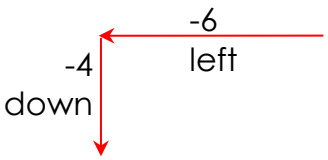
T (2 , 0)

the origin (0 , 0)



Exercise 3 **Complete**

The coordinates of
A = (-6, -4)

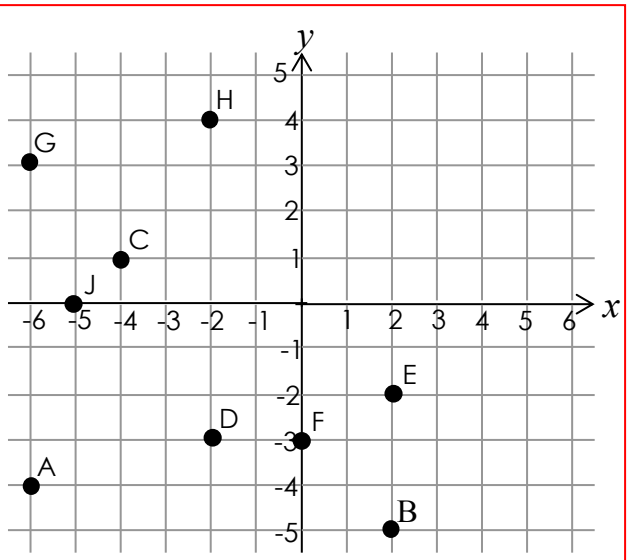


B = (2 , -5) C = (-4 , 1)

D = (___ , ___) E = (___ , ___)

F = (___ , ___) G = (___ , ___)

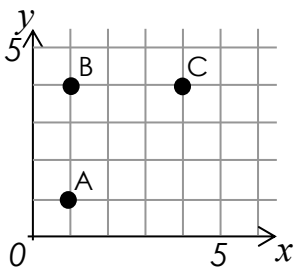
H = (___ , ___) J = (___ , ___)



Plot these points

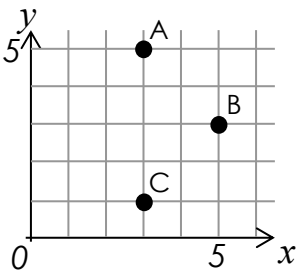
P (4 , -3) Q (-5 , 2) R (-2 , 1) S (-3 –5) T (6 , -5) U (-6 , -2)

Exercise 4 Complete

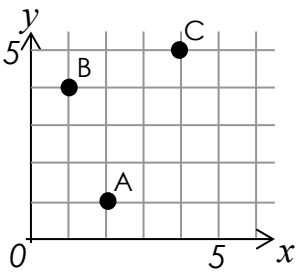


ABCD is a **square**

D= (___ , ___)

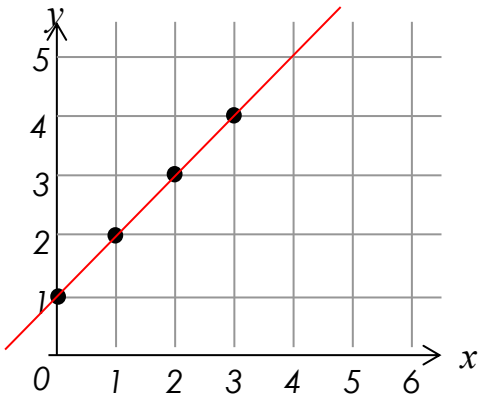


D= (___ , ___)



D= (___ , ___)

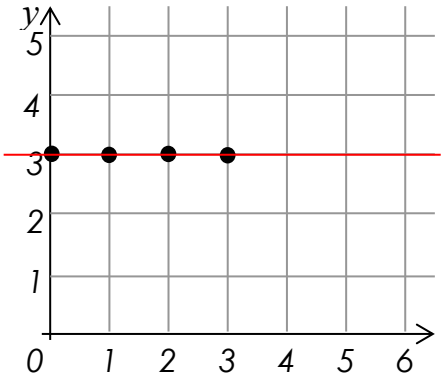
Points in straight lines



(x , y)
(0 , 1)
(1 , 2)
(2 , 3)
(3 , 4)

+1

formula: $y = x + 1$

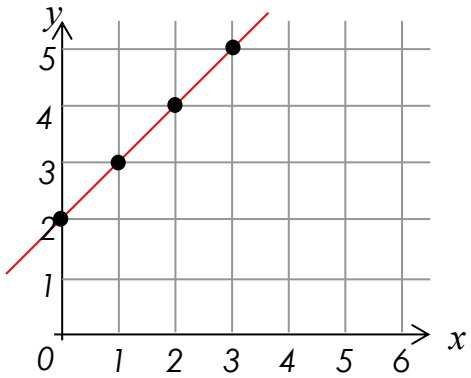


(x , y)
(0 , 3)
(1 , 3)
(2 , 3)
(3 , 3)

y is always 3

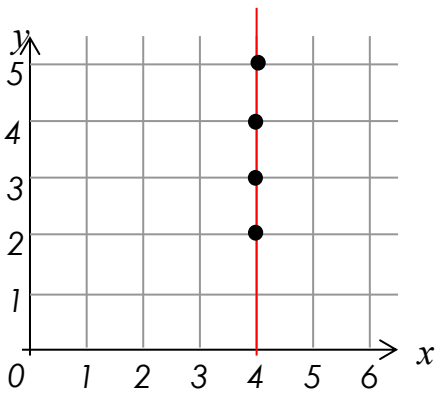
formula: $y = 3$

Exercise 5 Complete



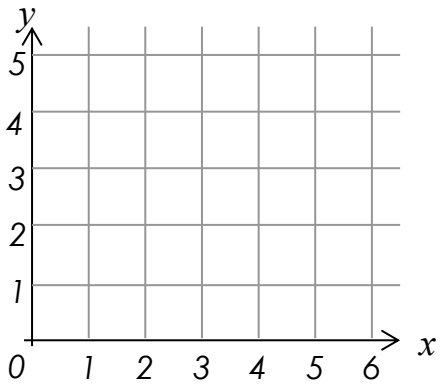
(x , y)
(___ , ___)
(___ , ___)
(___ , ___)
(___ , ___)

formula: $y =$ _____



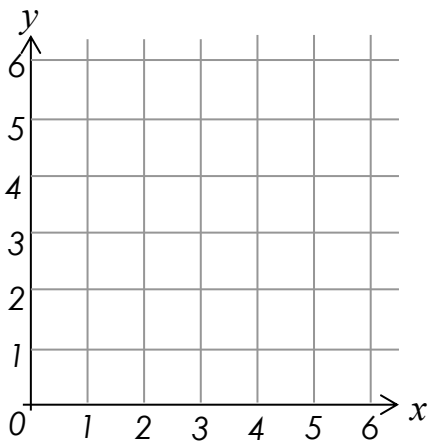
(x , y)
(___ , ___)
(___ , ___)
(___ , ___)
(___ , ___)

formula: $x =$ _____



(x , y)
(1 , 0)
(2 , 1)
(3 , 2)
(4 , 3)

formula: $y =$ _____



(x , y)
(0 , ___)
(1 , ___)
(2 , ___)
(3 , ___)

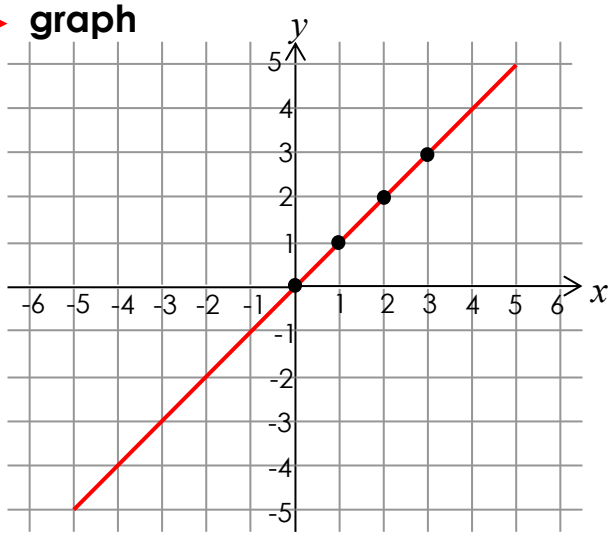
× 2

formula: $y =$ _____

A8 Linear Graphs

equation: $y = x$ → **table** → **graph**

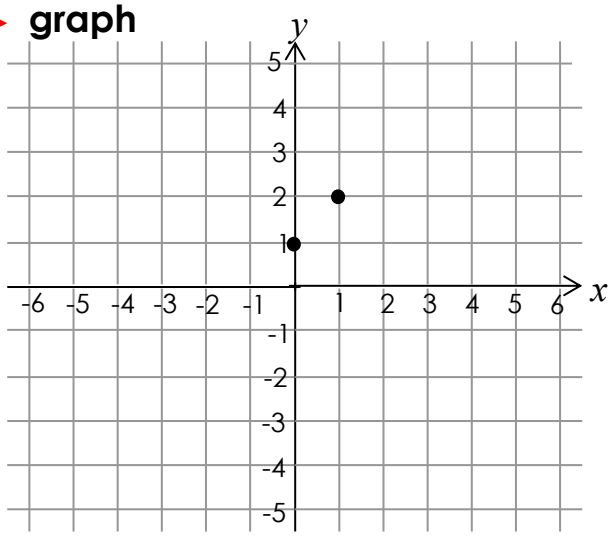
(x, y)
(0, 0)
(1, 1)
(2, 2)
(3, 3)



Exercise 1 **Complete**

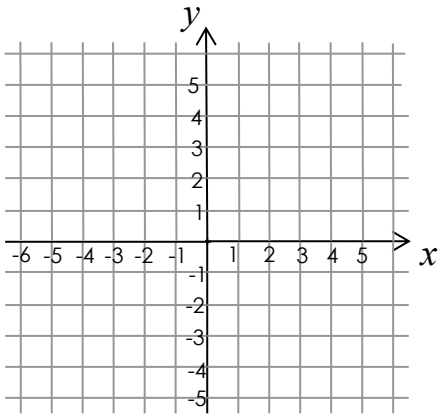
equation: $y = x + 1$ → **table** → **graph**

(x, y)
(0, 1)
(1, 2)
(2, __)
(3, __)



equation: $y = 2x$ → **table** → **graph**

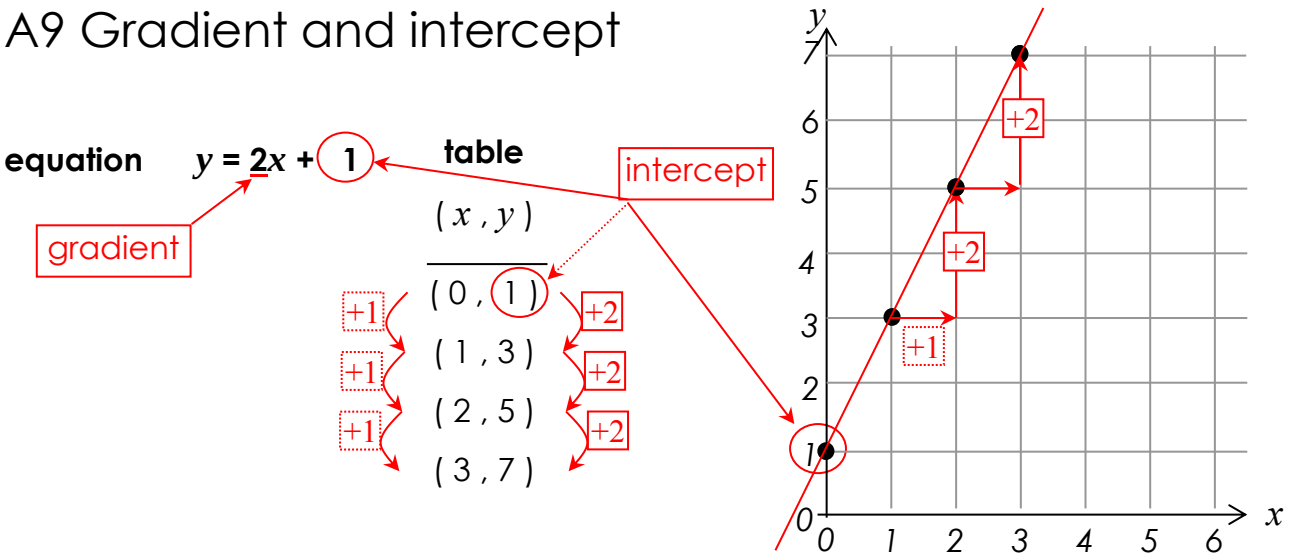
(x, y)
(0, 0)
(1, 2)
(2, __)
(3, __)



equation : $y = 3x + 2$ → **table** → **graph**
(complete a table and plot the graph, *use the worksheet Linear Graphs*)

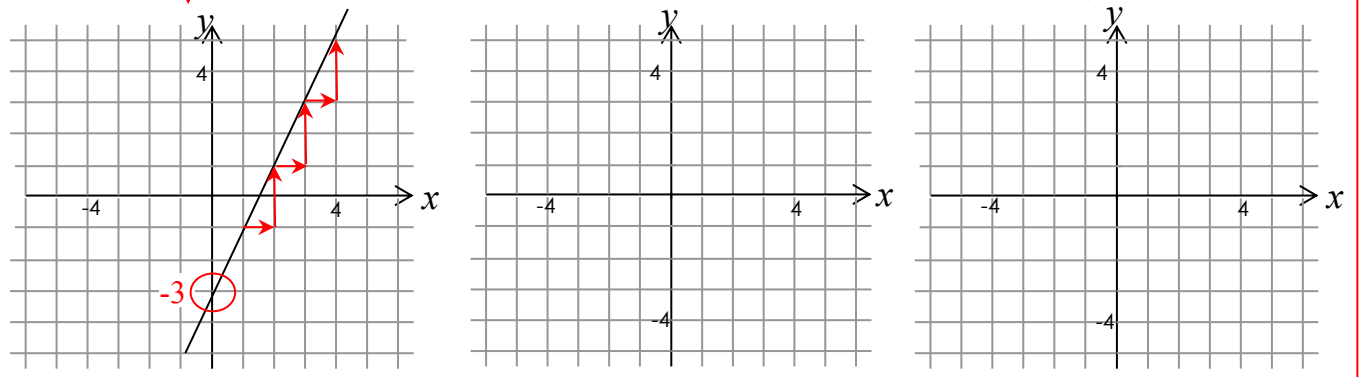
equation : $y = 4x - 3$ → **table** → **graph**
(complete a table and plot the graph, *use the worksheet Linear Graphs*)

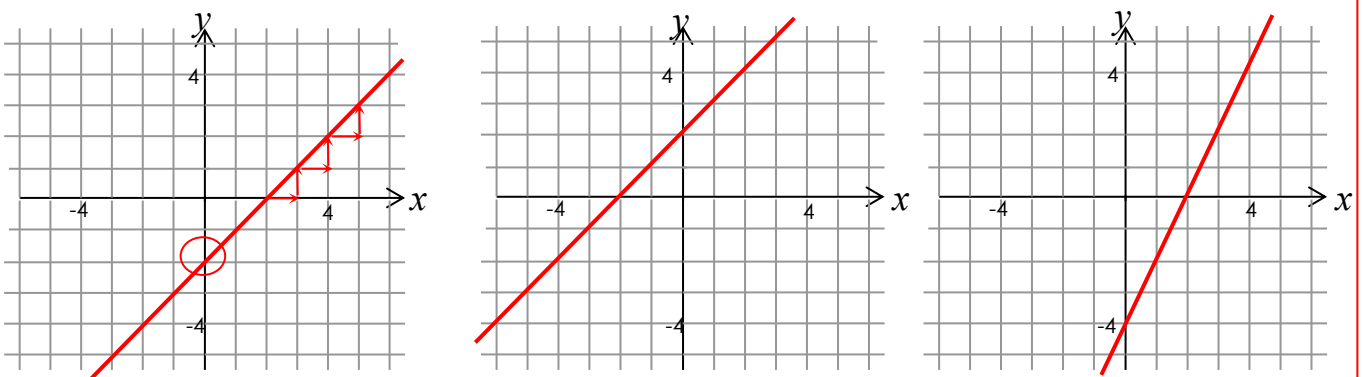
A9 Gradient and intercept



Exercise 1 Complete

equation: $y = 2x - 3$ $\quad y = 2x + 1$ $\quad y = 4x - 5$





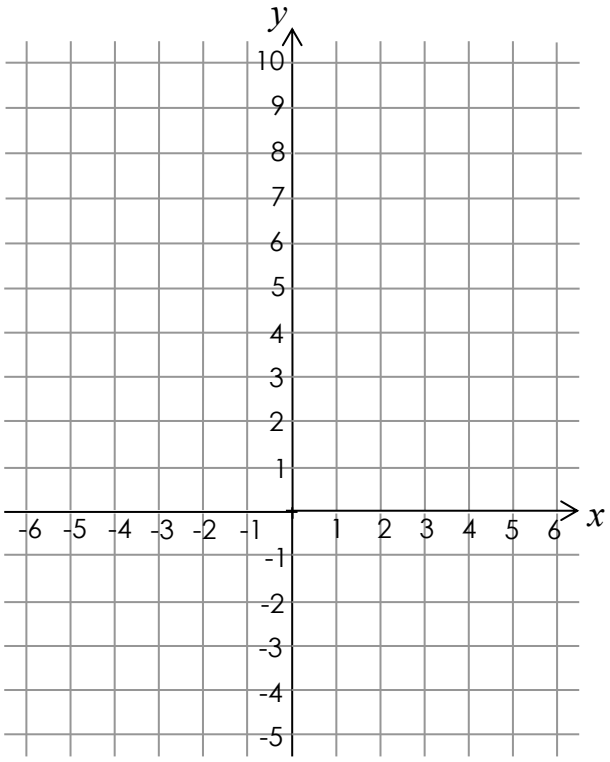
equation: $y =$ _____ $\quad$ _____ $\quad$ _____

Exercise 2 Complete

$y = 3x + 2$

equation: $y = 3x + 2$ ⟶ table ⟶ graph

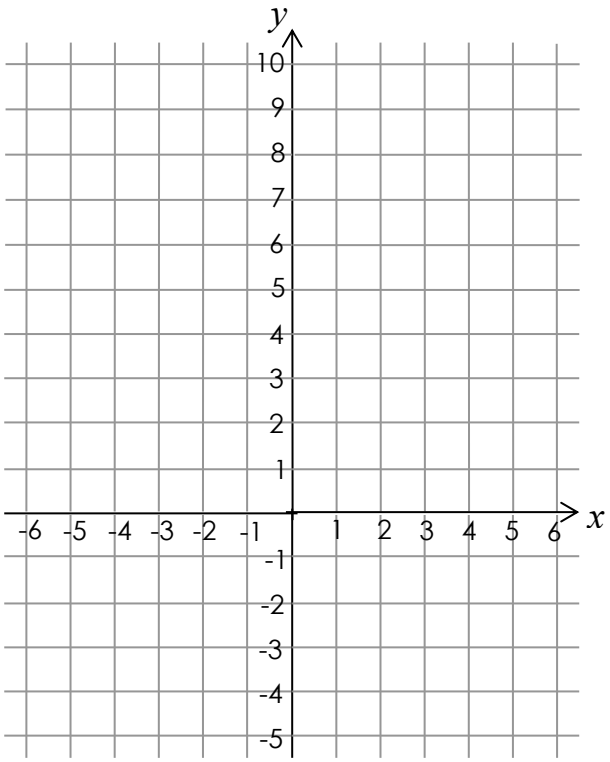
(x, y)
(0 , __)
(1 , __)
(2 , __)
(3 , __)



$y = 4x - 3$

equation: $y = 4x - 3$ ⟶ table ⟶ graph

(x, y)
(0 , __)
(1 , __)
(2 , __)
(3 , __)



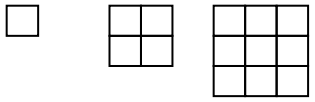
A12 Number Sequences

Exercise 1 **Complete**

	first	second	third	fourth	fifth	sixth	seventh	eighth	ninth	tenth
even numbers	2	4	6	8	10	12	.....	.....	.....	.....
odd numbers	1	3	5	7	9	11	.....	.....	.....	.....

Exercise 2 **Complete**

square numbers 1 4 9 -------- --------

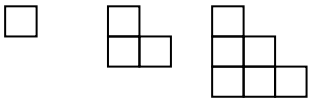


the **second** square number is 4

the **fourth** square number is _____

the **tenth** square number is _____

triangle numbers 1 3 6 -------- --------



the **second** triangle number is 3

the **fourth** triangle number is _____

the **tenth** triangle number is _____

Continue these **sequences:**

multiples of 3: 3 6 9 ___ ___ ___ ___

multiples of 9: ___ ___ ___ ___ ___ ___ ___ ___

powers of 2 2 4 8 16 ___ ___ ___ ___

powers of 10 ___ ___ ___ ___ ___ ___ ___ ___

the fourteenth (14^{th}) multiple of 2 is _____ the hundredth multiple of 9 is _____

the eleventh power of 2 is _____ the fifteenth power of ten is _____

Exercise 3 Complete

1) 4 7 10 ___ ___ ___ second **term** = 7 tenth term = ___

2) 98 196 194 ___ ___ ___ tenth term = ___ hundredth term = ___

3) 0.2 0.4 0.6 ___ ___ ___ tenth term = ___ hundredth term = ___

4) 0.3 0.5 0.7 ___ ___ ___ tenth term = ___ hundredth term = ___

5) 5 1 -3 ___ ___ ___ tenth term = ___ hundredth term = ___

n^{th} term

n	1	2	3	4	5	6	.....	100	.....
$2n + 5$	7	9	11	13	15	17		205	

$n = 1$
so $2n + 1 = 2 \times 1 + 1 = 7$

$n = 100$
so $2n + 1 = 2 \times 100 + 1 = 205$

Exercise 4

n^{th} term

$3n - 1$ 2 5 ___ ___ ___ hundredth term = _____ ($n = 100$)

$4n + 1$ ___ ___ ___ ___ ___ hundredth term = _____ ($n = 100$)

$10n - 9$ ___ ___ ___ ___ ___ hundredth term = _____ ($n = 100$)

$100 - n$ ___ ___ ___ ___ ___ hundredth term = _____ ($n = 100$)

$n^2 - 1$ ___ ___ ___ ___ ___ hundredth term = _____ ($n = 100$)

Finding the n^{th} term

+5 +5 +5 +5

4 9 14 19 24 .....

n^{th} term

$5n - 1$ $(5 - 1 = 4)$

Exercise 5 Complete

1) 2 4 6 8 ..... n^{th} term = _____ hundredth term = _____

2) 13 23 33 43 ..... n^{th} term = _____ hundredth term = _____

3) 6 13 20 27 ..... n^{th} term = _____ hundredth term = _____

4) 1 1.5 2 2.5 ..... n^{th} term = _____ hundredth term = _____

Exercise 6

Sequences and number patterns

x 2	
1	**2**
2	**4**
3	**6**
4	**8**
10	**20**
100	**200**
n	**$2n$**

The n^{th} term of the sequence 2,4,6,8..... is $2n$

x2	+1
1	**3**
2	**5**
3	**7**
4	**9**
10	**21**
100	**201**
n	**$2n + 1$**

The n^{th} term of the sequence 3,5,7,9,..... is $2n+1$

x3	
1	
2	
3	
4	
10	
100	
n	

x4	+3
1	7
2	
3	
4	
10	
100	
n	

1	
2	
3	
4	
10	
100	
n	**$4n-2$**

1	
2	
3	
4	
10	
100	
n	

The n^{th} term of the sequence _____ is $3n+5$

1	6
2	11
3	16
4	
10	
100	
n	

1	12
2	22
3	32
4	
10	
100	
n	

Exercise 7 Complete

1, 1, 2, 3, 5, 8, ____, ____, ____, ____,

a, a, $2a$, $3a$, $5a$, ____, ____, ____, ____,

a, b, $(a + b)$, _____, _____, _____, _____,

1, 4, 9, 16, ____, ____, ____, ____, The n^{th} term of this sequence is_____

2, 5, 10, 17, ____, ____, ____, ____, The n^{th} term of this sequence is_____

1, 8, 27, 64, ____, ____, ____, ____, The n^{th} term of this sequence is_____

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, ___ , ___ , ___ , ___ , The n^{th} term of this sequence is_____

R3 Ratio

3 grey = 2 black + 1 white

black : white

2:1

ratio

Exercise 1

black	white
2	1
	2
10	
	15
15	

x 5 … x 5

grey	black	white
30		
45		
75		
120		
141		

7 grey = 2 black + 5 white

black : white

2:5

ratio

Exercise 2

black	white
2	5
	20
	15
15	

x 3 … x 3

grey	black	white
35		25
42		
140		
357		

Equivalent ratios

2:1 = 4:2 = 6:3 = 8:4 = 100:50 = 72:36

2:3 = 4:6 = 6:9 = 8:12 = 50:75 = 24:36

Exercise 3

4:1 = 8 : ___ = 20 : ___ = ___ : 3 = ___ : 7

4:3 = 8:6 = 20 : ___ = ___ : 30 = ___ : 27

5:3 = 10:6 = 20 : ___ = ___ : 12 = ___ : 27

4:7 = 8 : ___ = 20 : ___ = ___ : 63 = ___ : 21

9 : 12 (÷3 / ÷3)

3 : ___ (x4 / x4)

12 : ___

4 : 12

___ : ___

___ : 18

20 : 72

___ : ___

15 : ___

R4 Dividing in a ratio

Divide 35 in the ratio 2 : 3

2 : 3

×7

14 : 21

2 + 3 = 5

35 ÷ 5 = 7

(14 + 21 = 35)

Exercise 4

Divide 90 in the ratio 4 : 5

4 : 5

___ + ___ = ___

_____

(_____)

_____

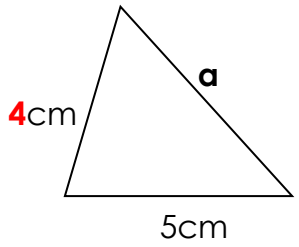
Divide 65 in the ratio 2 : 3 _____

Divide 140 in the ratio 4 : 3 _____

Divide 900 in the ratio 2 : 3 : 4 _____

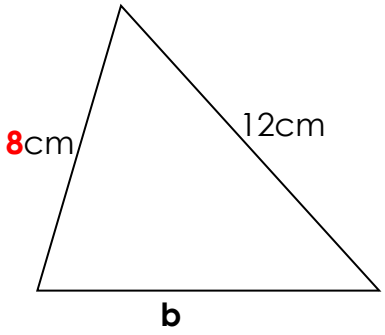
Divide 80 in the ratio 2 : 3 : 5 _____

Enlargement (congruent triangles)



x 2

÷2

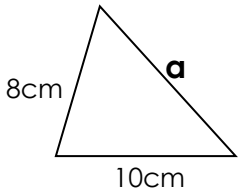


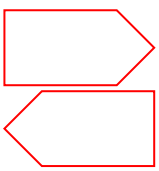
8 ÷ **4** = 2

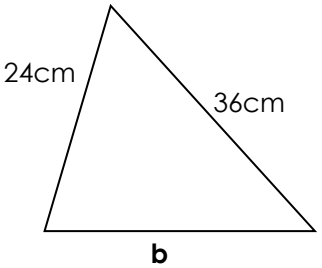
b = 5 x 2 = 10 cm

a = 12 ÷ 2 = 6 cm

Exercise 5

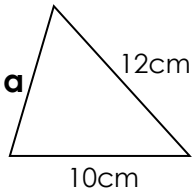


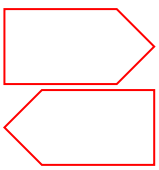


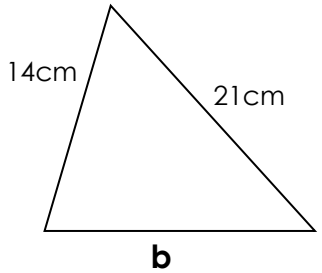


a = _____

b = _____





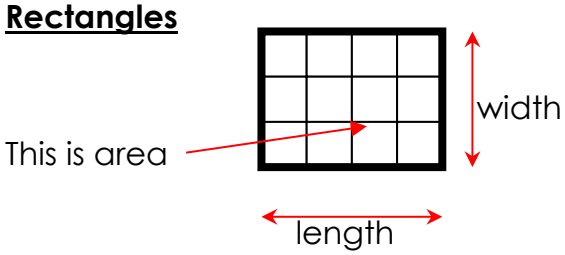


a = _____

b = _____

G1 Area and Perimeter and Volume

Rectangles

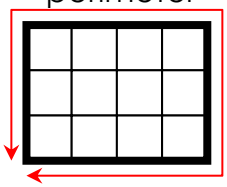


the width is 3 cm
the length is 4 cm
the area is 12 cm^2

units

length x width = area

perimeter

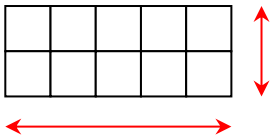


The perimeter is 14cm

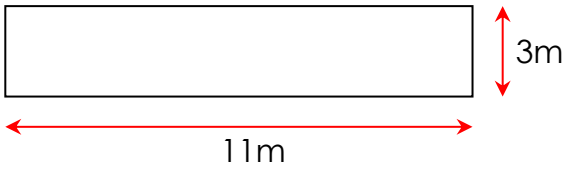
Exercise 1

Area of a rectangle

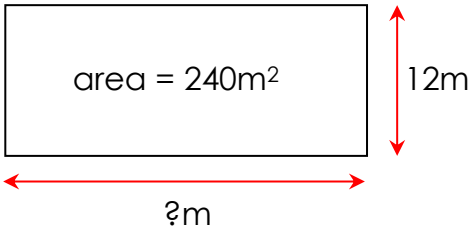
length x width = area



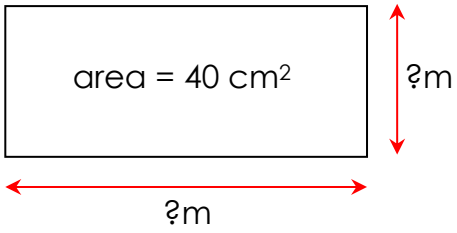
the width is _____ cm
the length is _____ cm
the area is _____ cm^2
the perimeter is _____cm



the _____is _____ m
the _____is _____ m
the _____is _____ m^2
the _____is _____ m

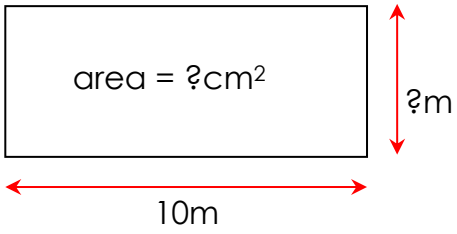


the _____is _____ m
the _____is _____ m
the _____is _____ m^2
the _____is _____ m



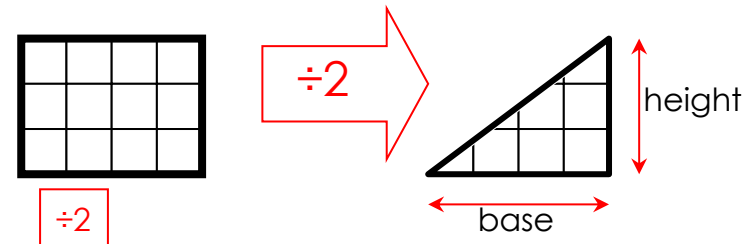
the _____is _____ cm
the _____is _____ cm
the _____is _____ cm^2
the perimeter is 26 cm

the area = the perimeter



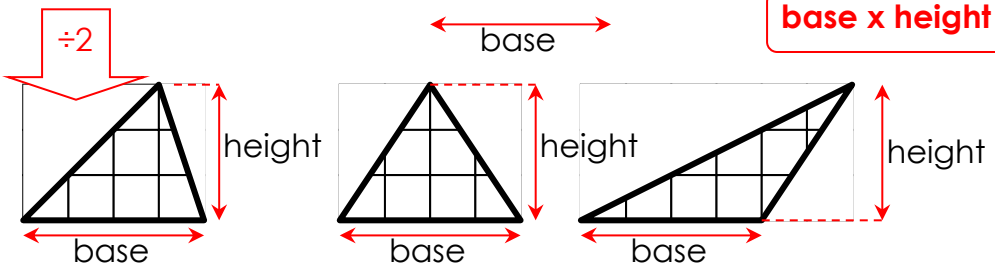
the _____is _____ cm
the _____is _____ cm
the _____is _____ cm^2
the _____is _____ cm

Triangles



For all 4 triangles
the height is 3 cm
the base is 4 cm
the area is 6cm²

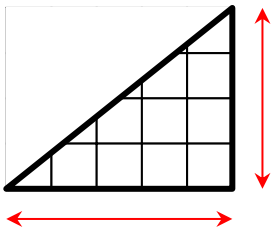
base x height ÷ 2 = area



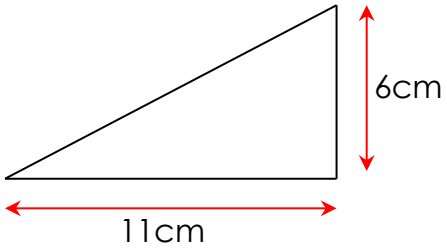
Exercise 2

Area of triangles

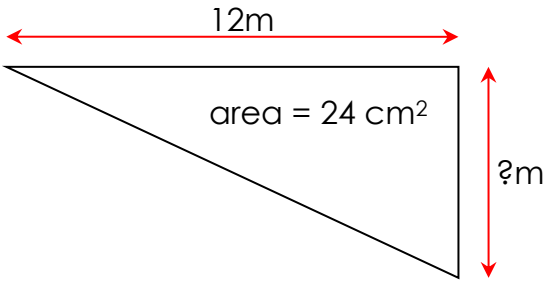
base x height ÷ 2 = area



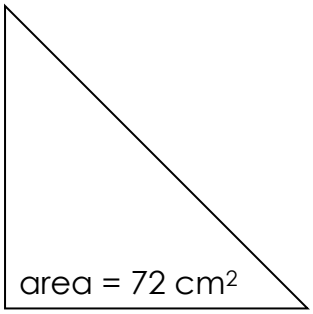
the height is _____cm
the base is _____ cm
the area is _____cm²



the _____is_____cm
the _____is_____cm
the _____is_____cm²



the _____is_____m
the _____is_____m
the ____area____is__24__m²

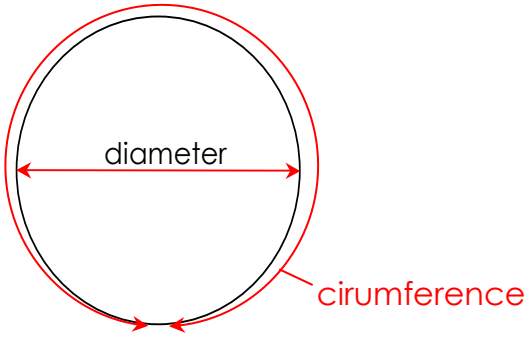


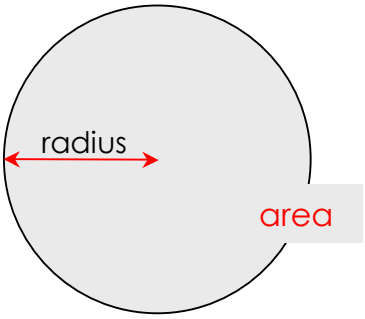
the base = the height

the _____is_____cm
the _____is_____cm
the ____area____is__72__cm²

Area and Circumference of a Circle

$\pi = 3.14159265358979323846264338327 9.....$





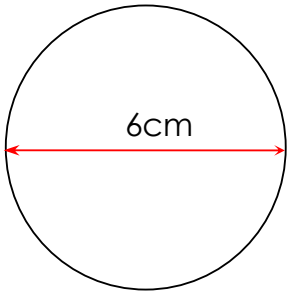
circumference = π x diameter

C = π x *d*

area = π x radius squared

A = π x r^2

Exercise 3 Complete

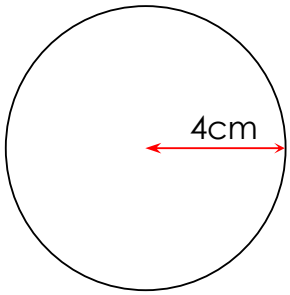


diameter = _____cm

circumference = ***π x d*** = π x 6 = 18.8 cm

radius = _____cm

area = ***π x r^2*** = π x 3^2 = _____cm^2

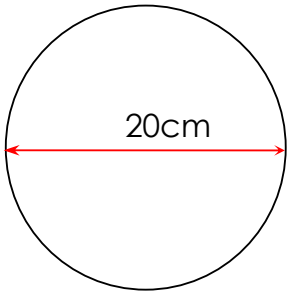


diameter = _____cm

circumference = _____cm

radius = _____cm

area = _____cm^2

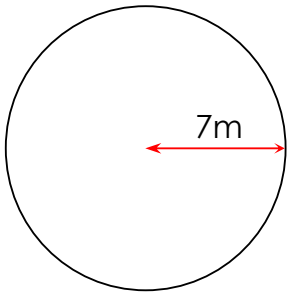


diameter = _____

circumference = _____

radius = _____

area = _____



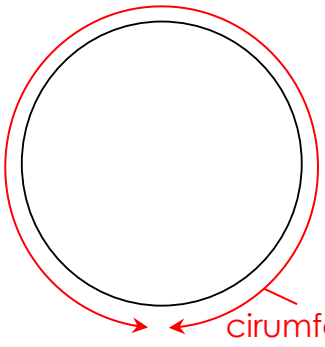
diameter = _____

circumference = _____

radius = _____

area = _____

Exercise 4 Complete

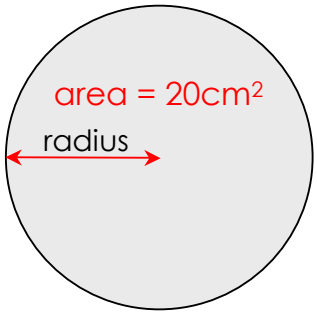


circumference = 628m

diameter = _____

radius = _____

area = _____



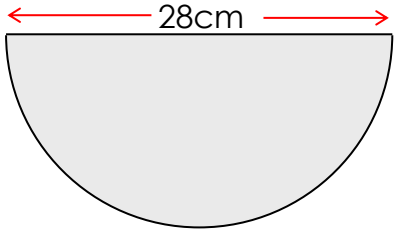
area = 20cm^2

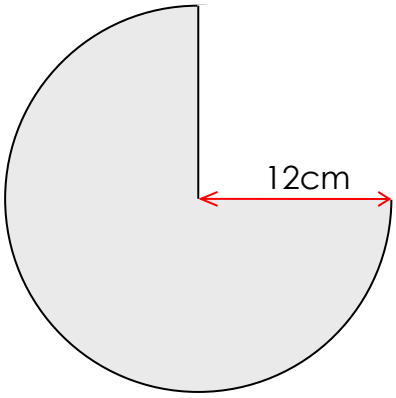
radius = _____

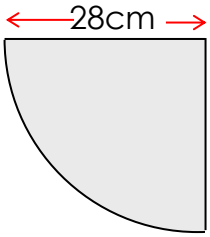
diameter = _____

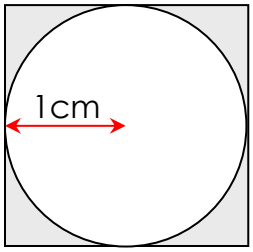
circumference = _____

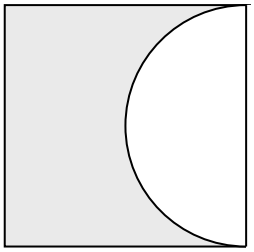
Exercise 5 Calculate the grey areas

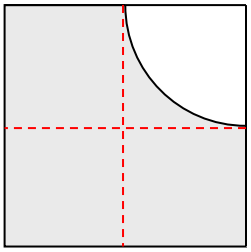








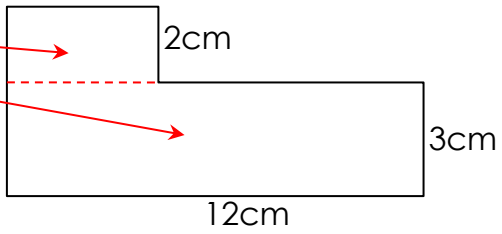




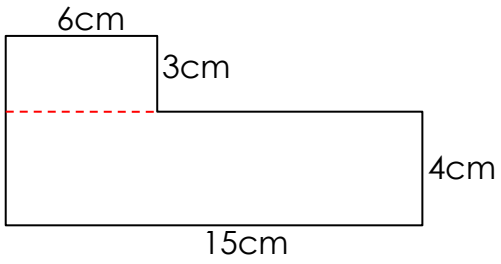
Adding areas

area 1 $2 \times 5 = 10cm^2$
area 2 $3 \times 12 = 36cm^2$

total area = area 1 + area 2
total area = 10 + 36 = 46 cm^2

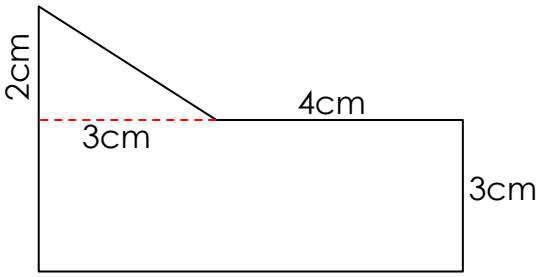


Exercise 6 **Complete**



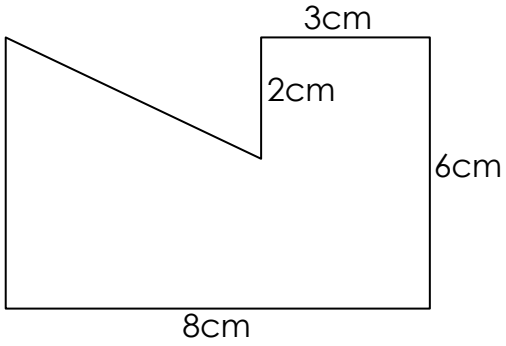
area 1 _____
area 2 _____

total area = area 1 + area 2
total area = _____



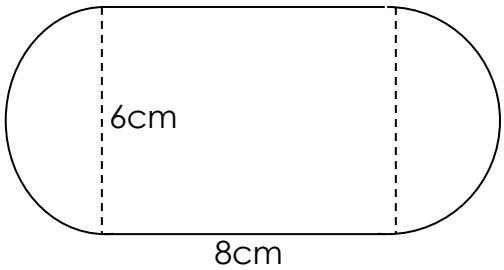
area 1 _____
area 2 _____

total area = _____



area 1 _____
area 2 _____
area 3 _____

total area = _____

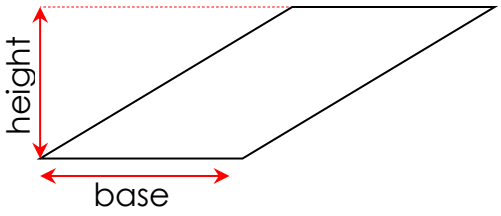


_____
_____
_____

total area = _____

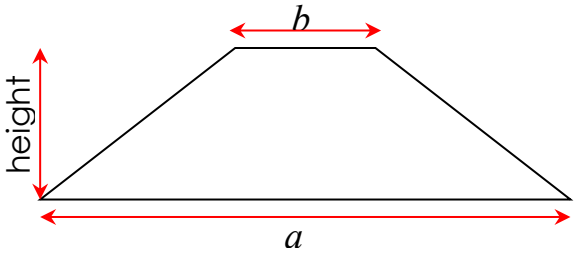
Area of a parallelogram and trapezium

Parallelogram



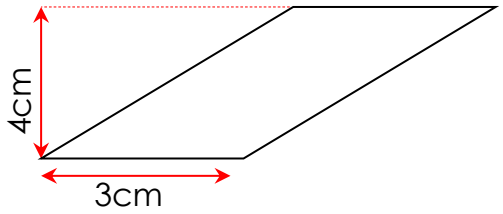
area = base x height

Trapezium



area = height x (a + b) ÷ 2

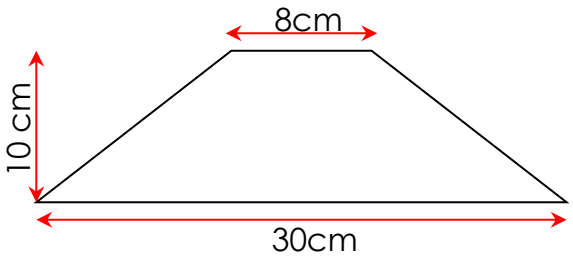
Exercise 7 **Complete**



shape parallelogram

height _____ base _____

area = base x height = _____

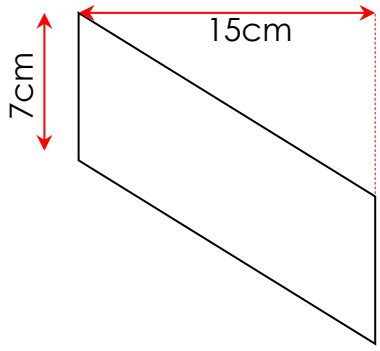


shape _____

height _____

a _____ b _____

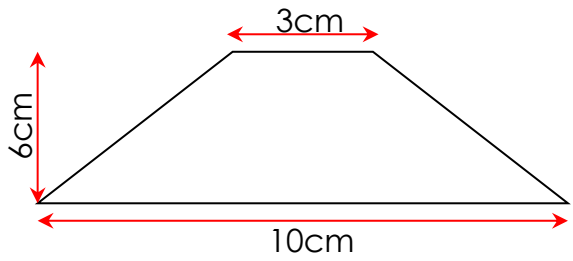
area = _____ = _____



shape _____

height _____ base _____

area = _____ = _____



shape _____

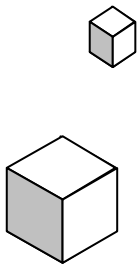
height _____

a _____ b _____

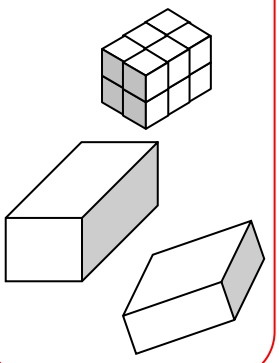
area = _____ = _____

Volume

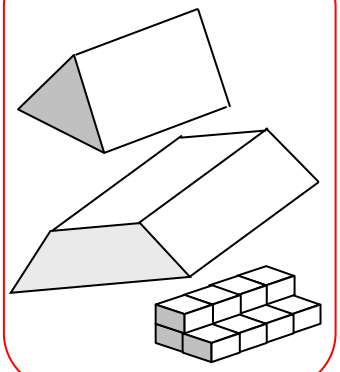
Cubes



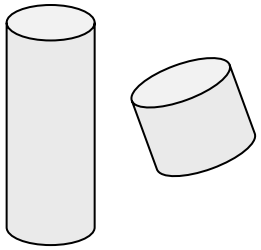
Cuboids



Prisms

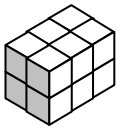


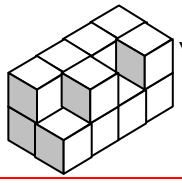
Cylinders



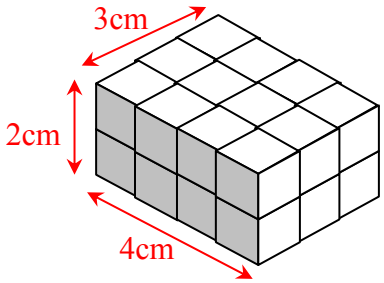
Exercise 8 **Complete**

 volume = 1cm^3

 volume = 12cm^3

 volume = _____cm^3

Exercise 9 **Complete**

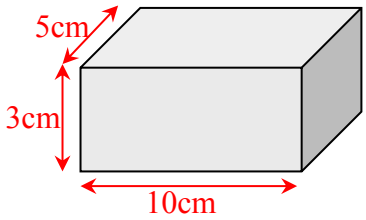


Volume of cubes and cuboids

length = 4cm width = 3cm height = 2cm

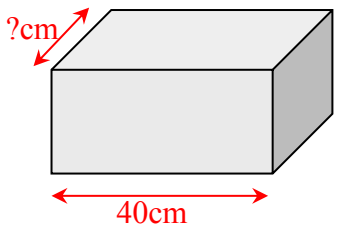
volume = length x width x height

volume = 4 x 3 x 2 = 24 cm^3



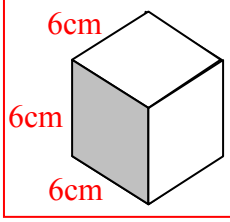
length = _____ width = _____ height = _____

volume = _____

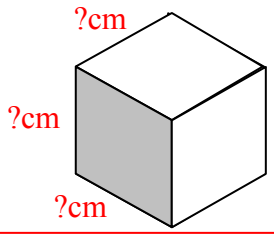


length = _____ width = _____ height = _____

volume = 72cm^2



volume = _____

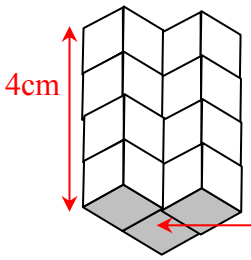


volume of the cube= 8000cm^3

length = _____

Exercise 10 Complete

Volumes of prisms

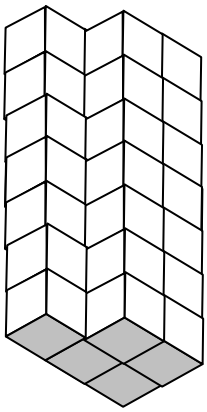


height = 4cm

volume = base area x height

volume = 3 x 4 = 12 cm^3

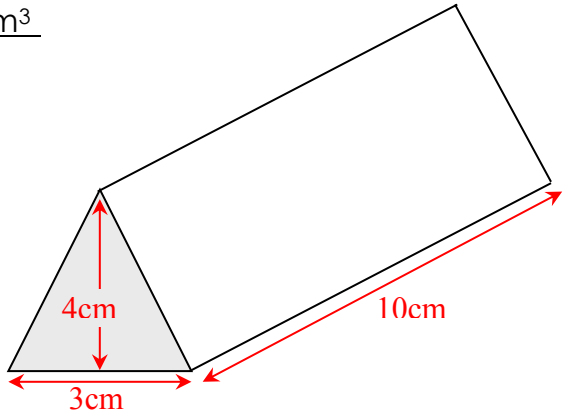
base area = $3cm^2$



height = _____

base area = _____

volume = _____

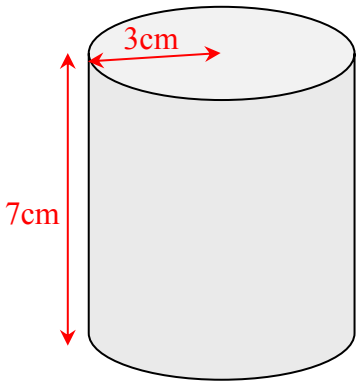


height = _____

base area = _____
(area of triangle)

volume = _____

Exercise 11 Complete

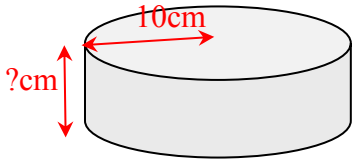


Cylinder = circular prism

height = _____

base area = _____
(area of circle = πr^2)

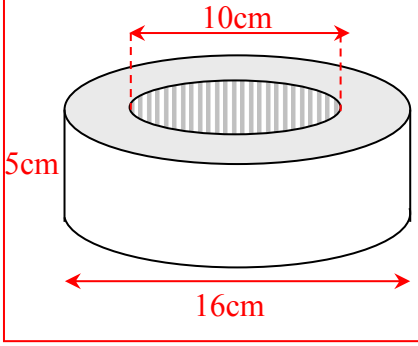
volume = _____



height = _____

base area = _____

volume = 314000cm^3



_____

_____

_____

volume = _____

G2 Constructions

A pair of compasses

Drawing triangles given three sides

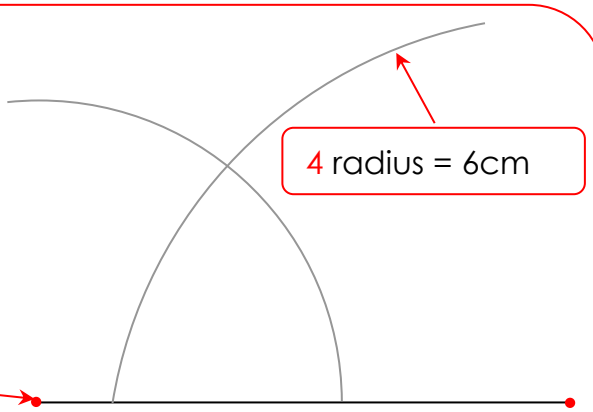
For example **4cm 6cm 7 cm**

1 draw the 7cm line

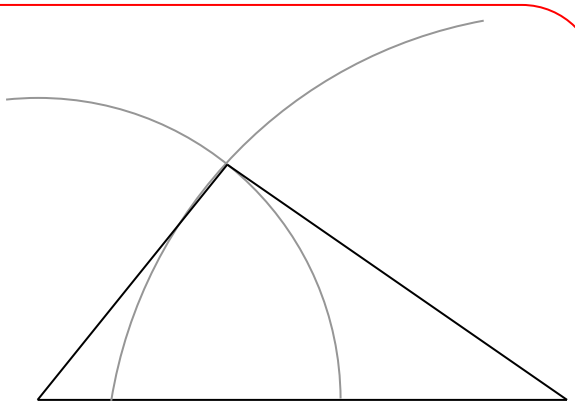
2 set compasses to radius = 4cm

4 radius = 6cm

3 centre compasses here



5 now draw the other two



Exercise 1 **Draw these triangles on paper**

1) 6cm, 3cm, 7cm

2) 6cm, 7cm, 8cm

3) 5cm, 5cm, 5cm

4) A triangle with perimeter = 18cm

5) A right angled triangle (use Pythagoras)

Drawing triangles given two sides and the angle between them

For example **5cm 10cm 37°**

Draw the 10cm line

↓

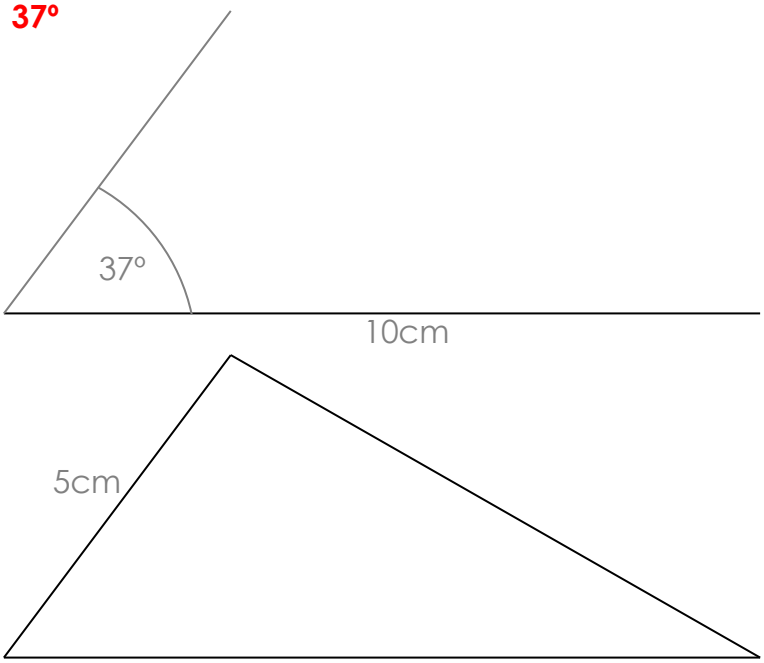
Measure the 37° angle

↓

Draw the 5cm line

↓

Draw the third side



Exercise 2 **Draw these triangles on paper**

1) 5cm, 6cm, 65° 2) 6cm, 3cm, 135° 3) 4cm, 4cm, right angled

Drawing triangles given a side and two angles

For example **8cm 45° 30°**

Draw the 8cm line

↓

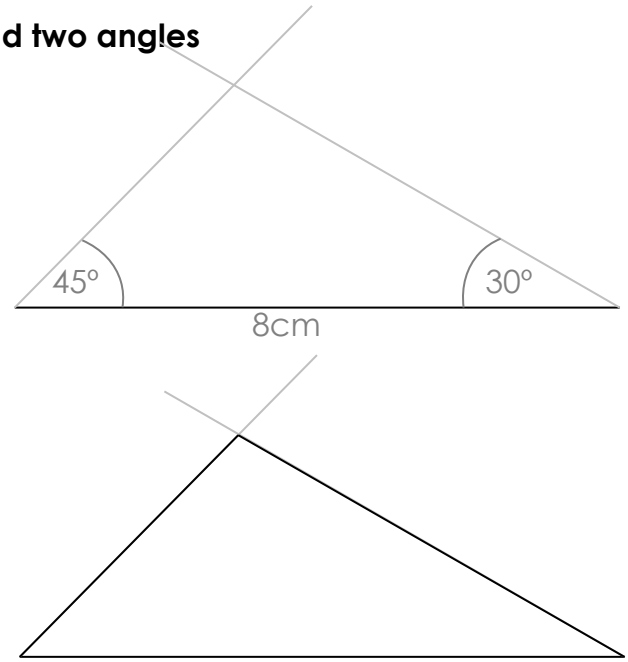
Measure the two angles

↓

Draw two lines

↓

Complete the triangle



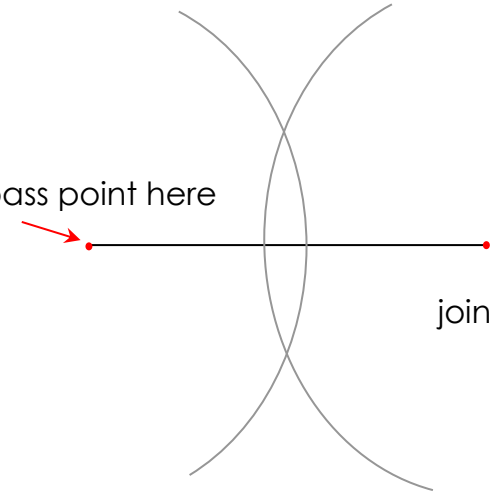
Exercise 3 **Draw these triangles on paper**

1) 6cm, 40°, 60° 2) 7cm, 28°, 120° 3) 5cm, 60°, 60°

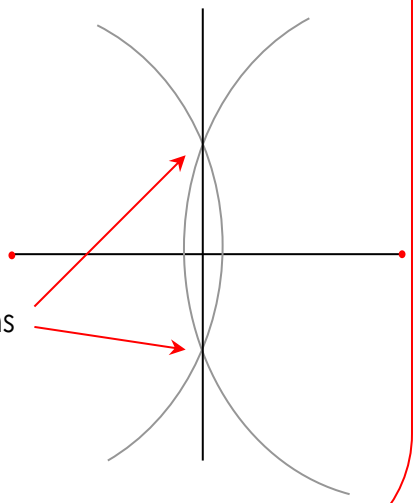
Perpendicular bisectors

Start with a line

set radius at more than half the line



place compass point here



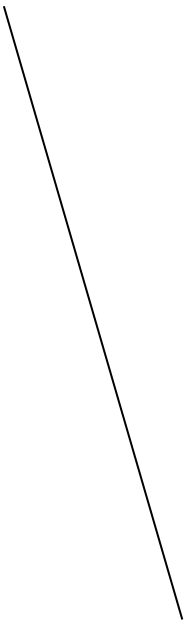
join the intersections

both circles must have the same radius

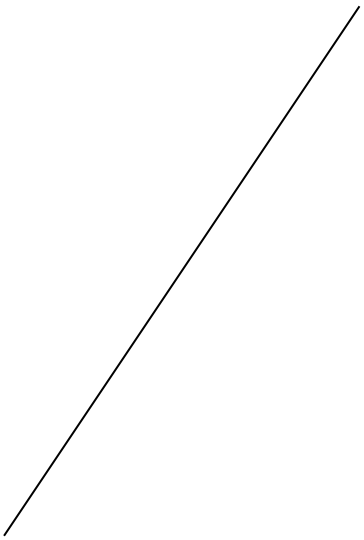
Perpendicular = at right angles (90°) **Bisector** = cuts in half (½)

Exercise 4 Construct perpendicular bisectors

2)

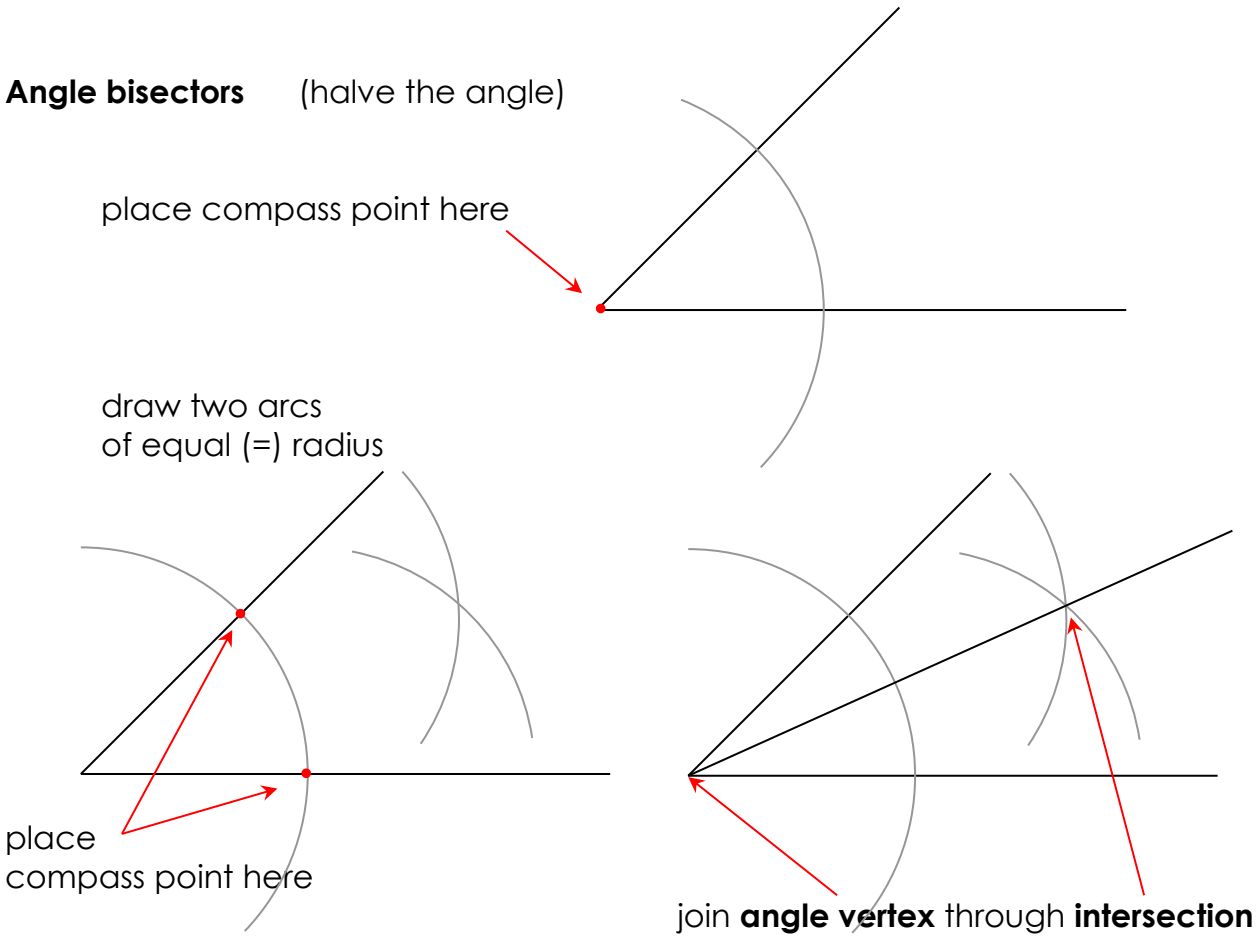


3)

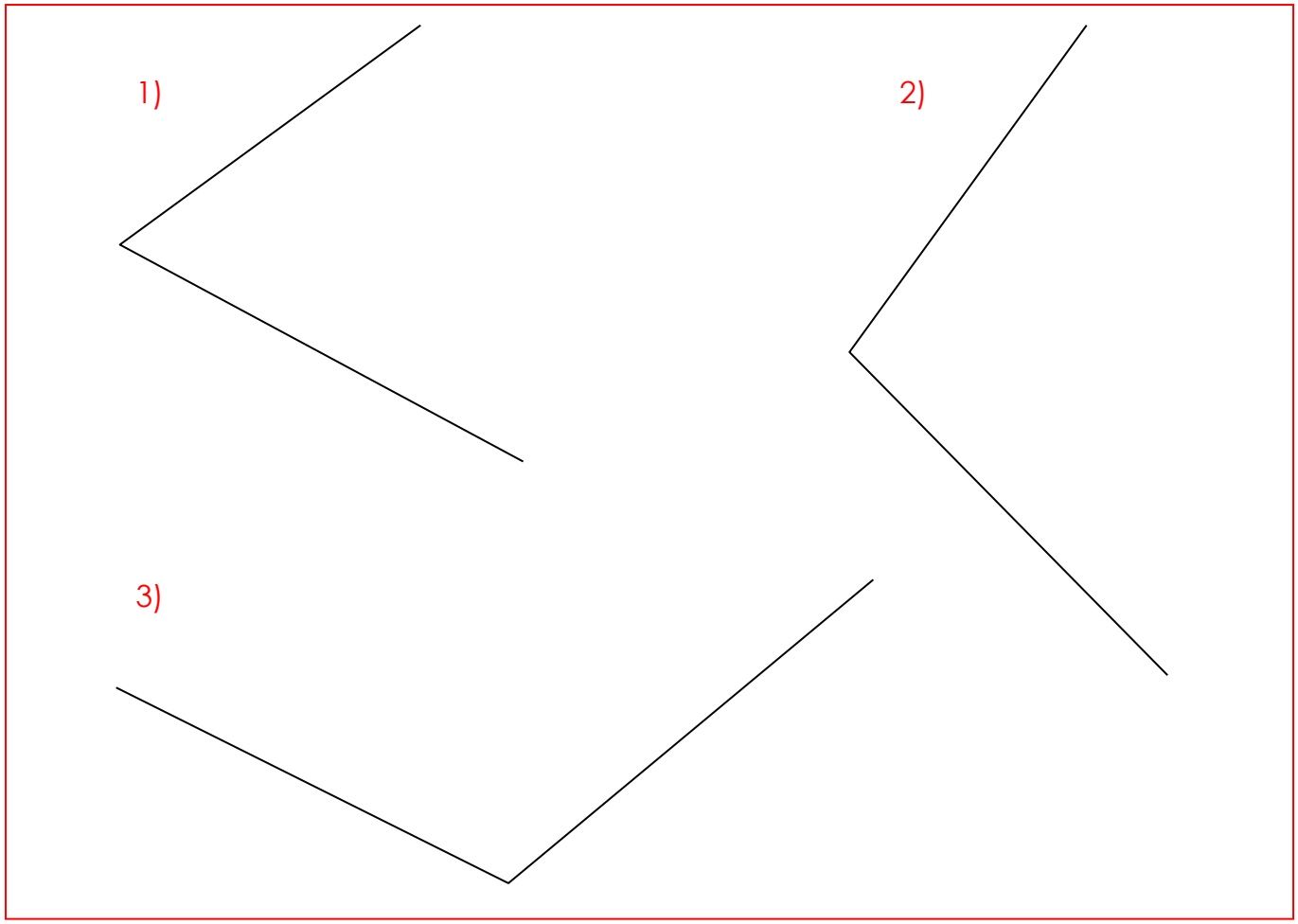


1)

Angle bisectors (halve the angle)

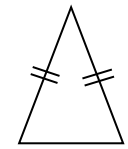


Exercise 5 **Bisect these angles**

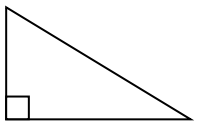


G5 Triangles and Quadrilaterals

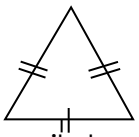
Triangles



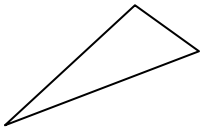
isosceles



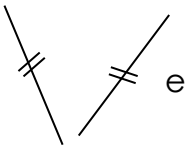
right angled



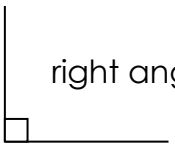
equilateral



scalene



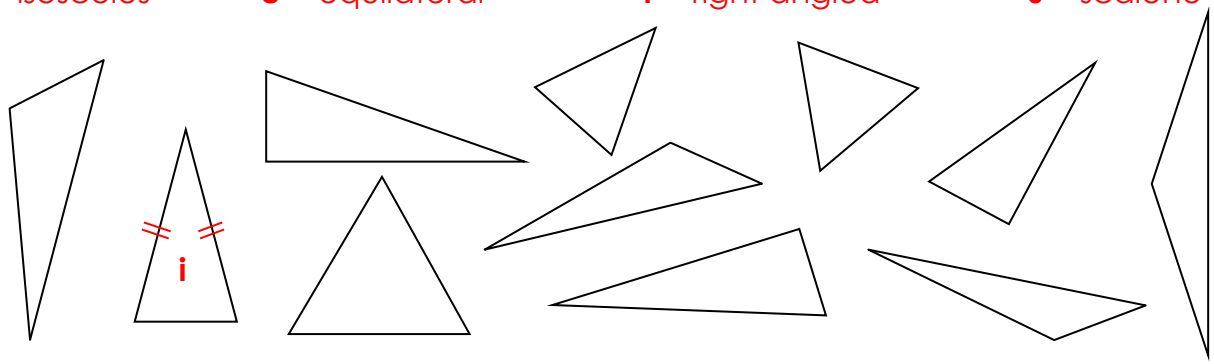
equal length



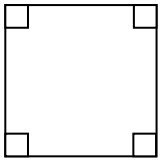
right angle (90°)

Exercise 1 **Label these triangles: mark right angles and equal sides**

i = isosceles **e** = equilateral **r** = right angled **s** = scalene

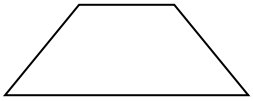


Quadrilaterals

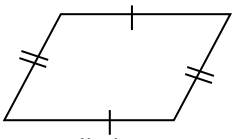


square

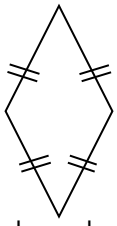
rectangle



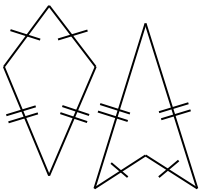
trapezium



parallelogram



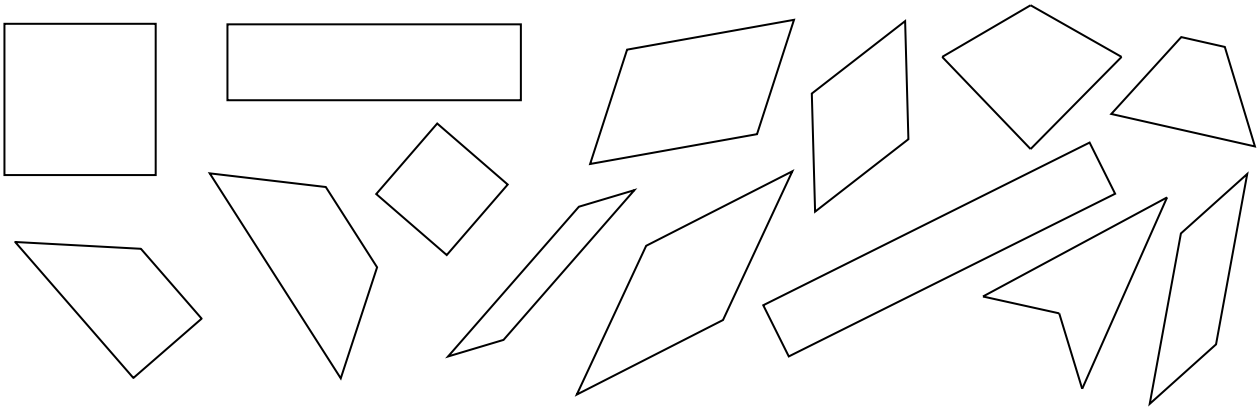
rhombus



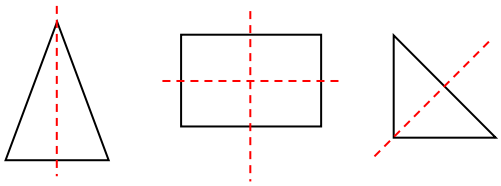
kite

Exercise 2 **Label these quadrilaterals: mark right angles and equal sides**

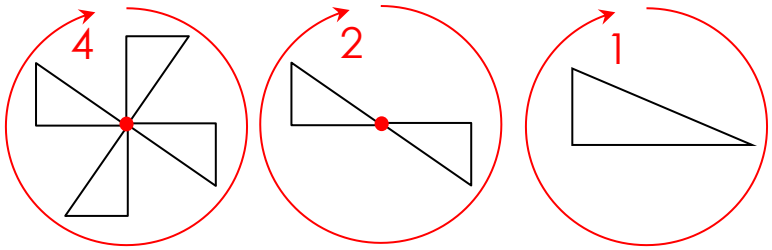
s = square **r** = rectangle **t** = trapezium **p** = parallelogram **rh** = rhombus **k** = kite



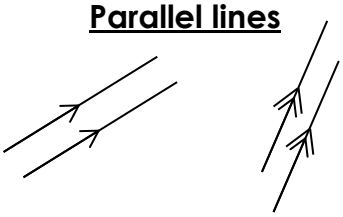
Axes (lines) of symmetry



Order of rotational symmetry

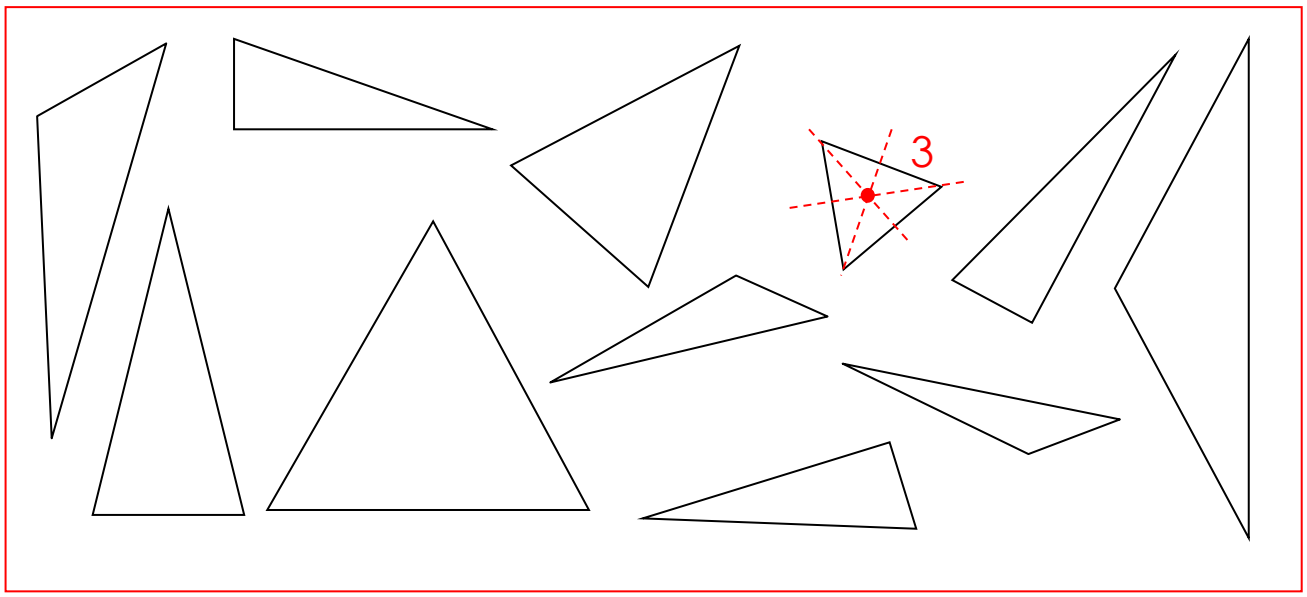


Parallel lines



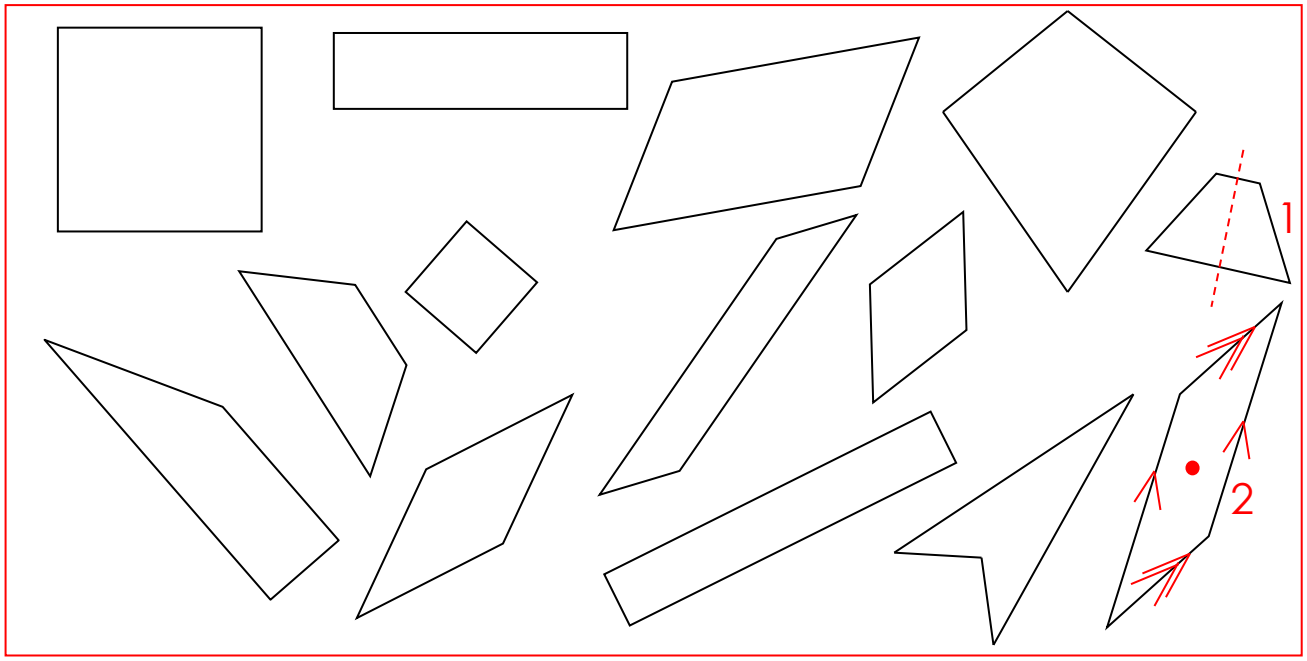
Exercise 3

Mark all the lines of symmetry and order of rotational symmetry on these triangles



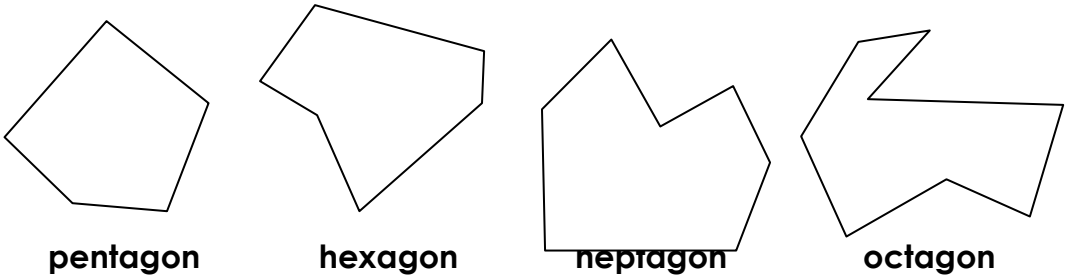
Exercise 4

Mark all the lines of symmetry, the order of rotational symmetry and parallel lines



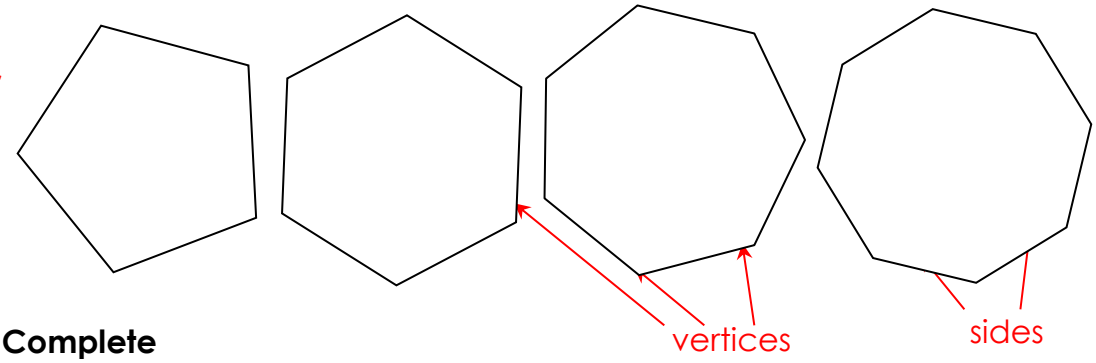
Polygons

Irregular



pentagon **hexagon** **heptagon** **octagon**

Regular



Exercise 5 **Complete**

A **pentagon** has five (5) **sides** and five **vertices**

A **hexagon** has _____ sides and _____ vertices

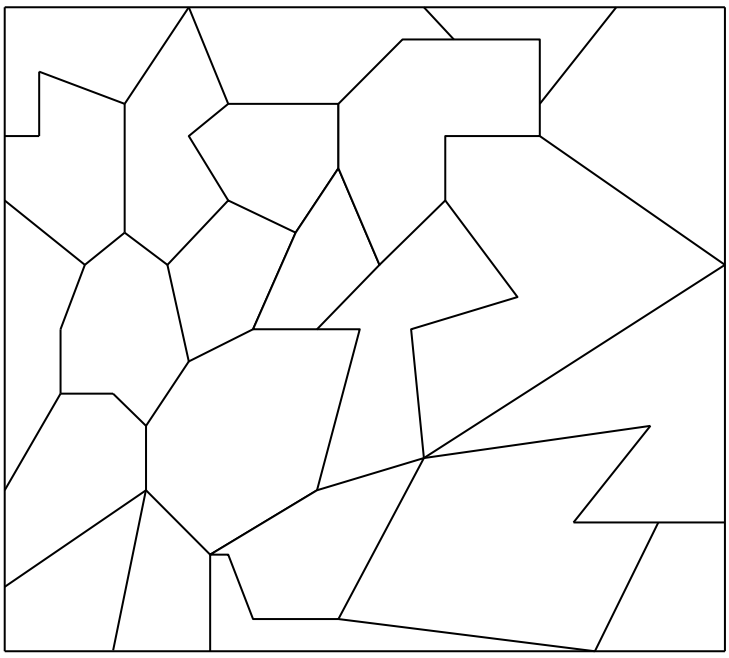
A _____ has seven sides and _____

An **octagon** has _____

A **triangle** _____

A **quadrilateral** _____

How many polygons are there in this diagram?



Exercise 6 **Complete**

There are three quadrilaterals

There are _____ pentagons

There are _____ hexagons

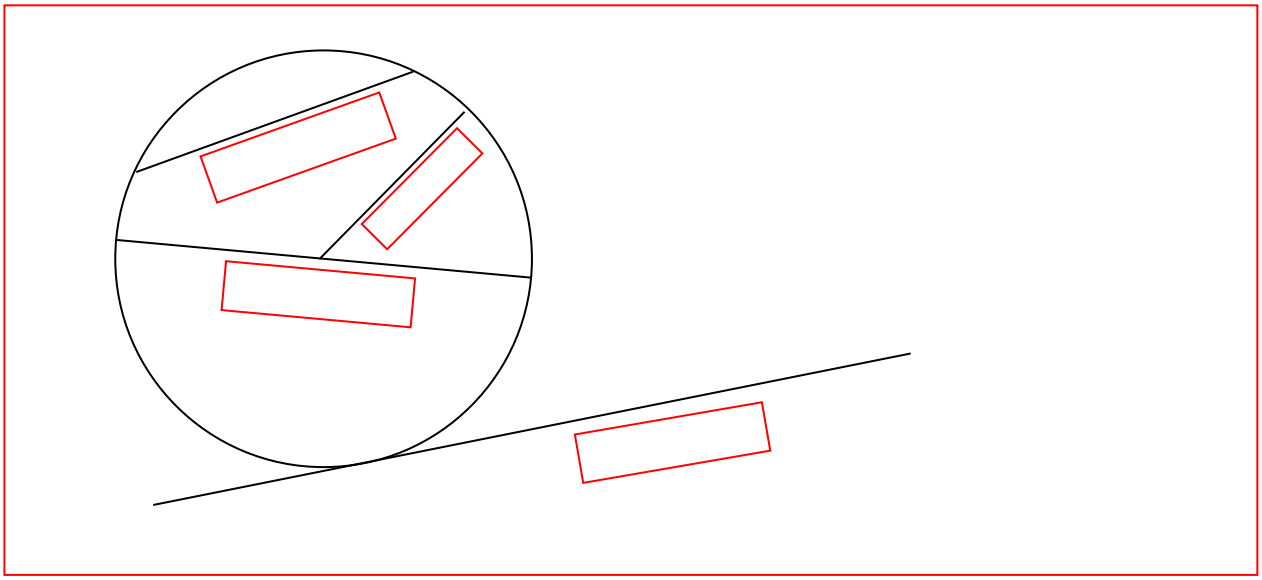
There are _____ heptagons

There are _____ octagons

There are _____ polygons

Circles

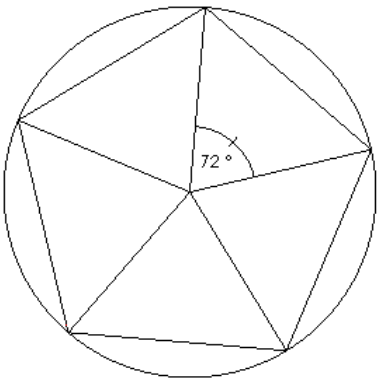
Exercise 7 **Write the names in the** ☐



Exercise 8 **Complete**

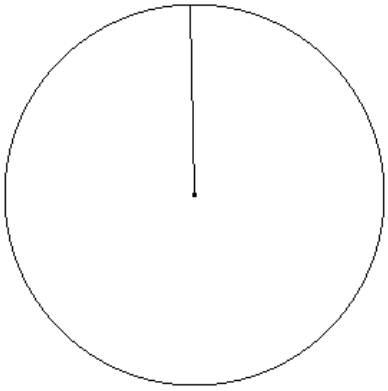
A regular pentagon

360 ÷ 5 = 72°



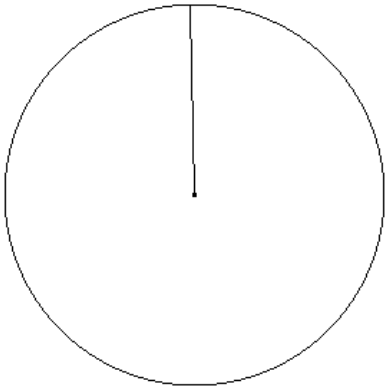
A regular hexagon

360 ÷ ____ = ____



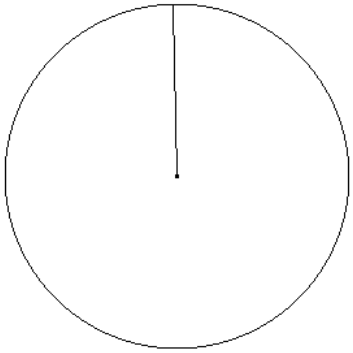
A regular octagon

360 ÷ ____ = ____



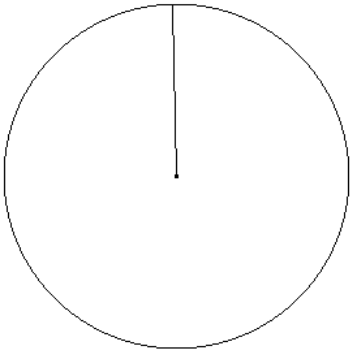
A regular heptagon

360 ÷ ____ = ____



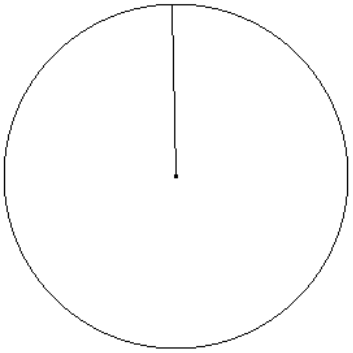
A regular nonagon

(nine sides)



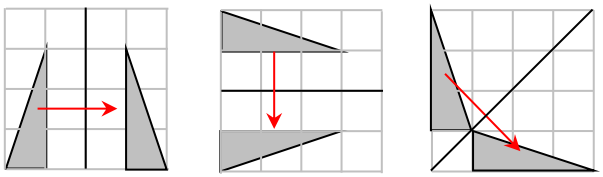
A regular decagon

(ten sides)

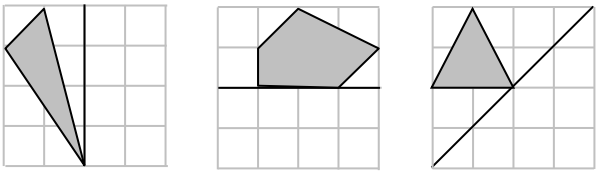


G6 Transformations

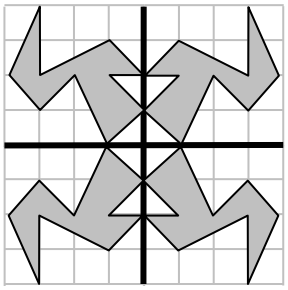
Reflection

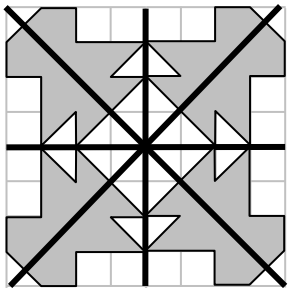


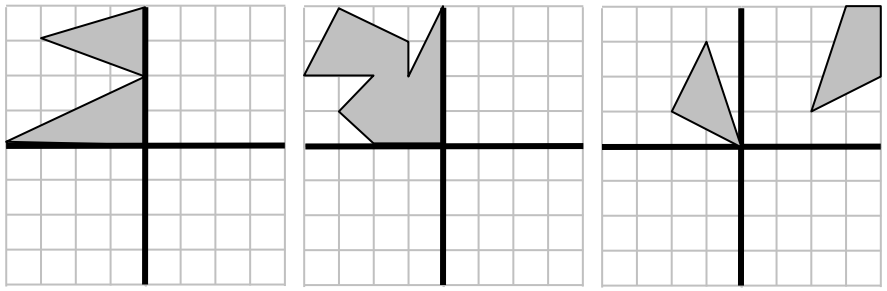
Exercise 1 **Complete the reflections**

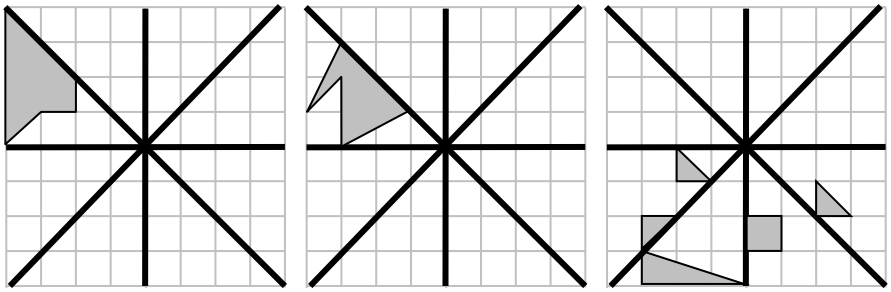


Exercise 2 **Complete these reflections**

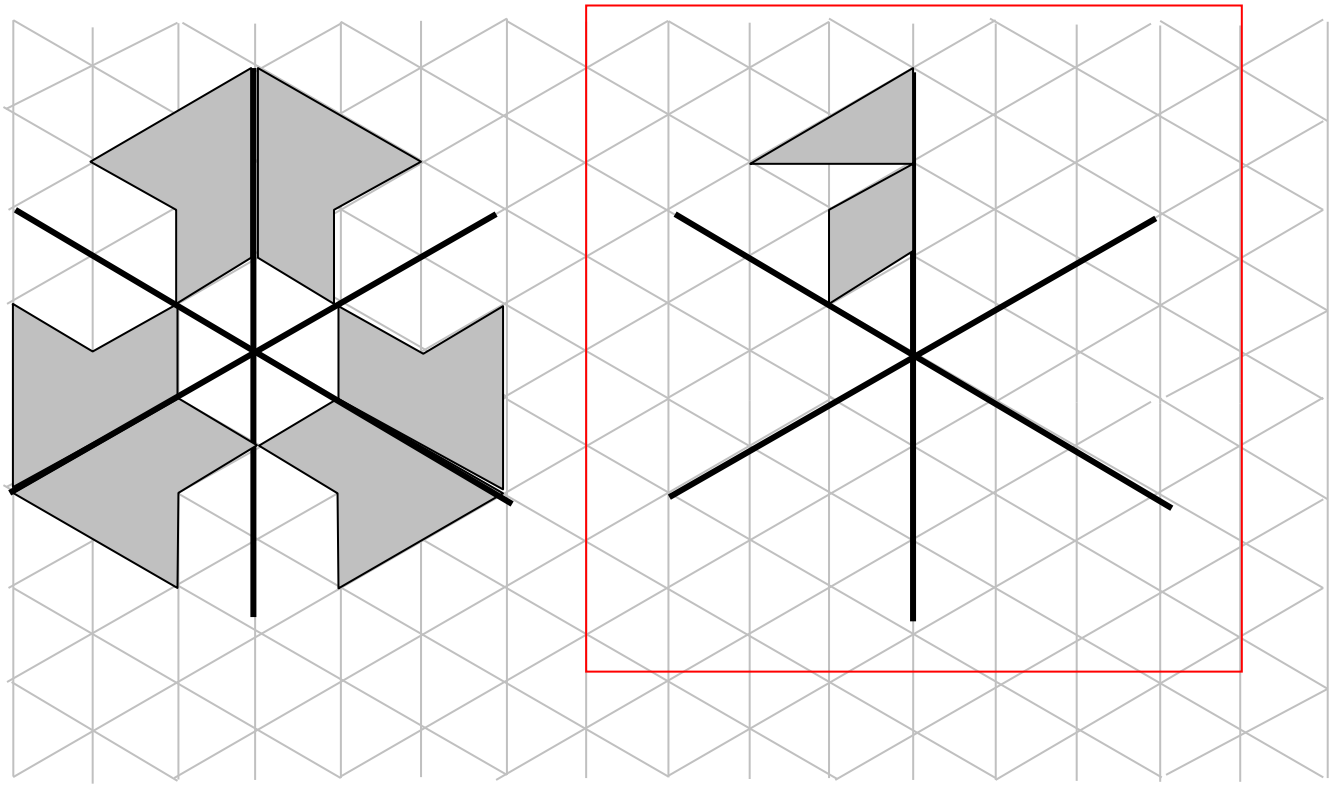




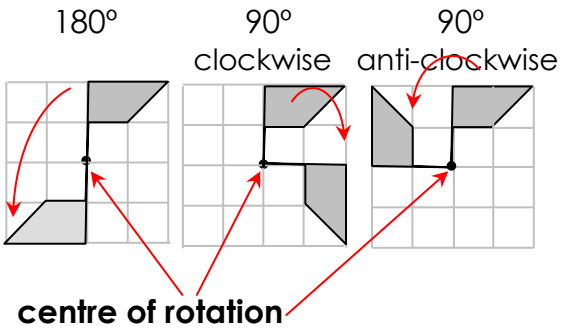




Exercise 3 **Complete this reflection**

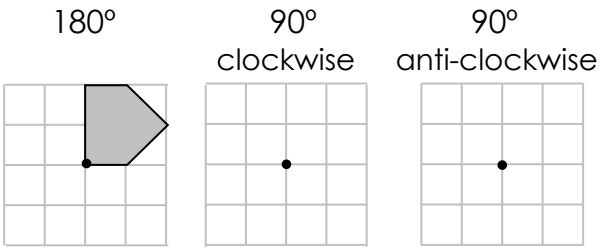


Rotation



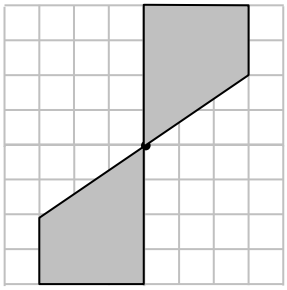
centre of rotation

Exercise 4 Complete these rotations

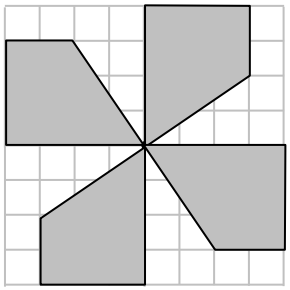


Patterns and order of symmetry

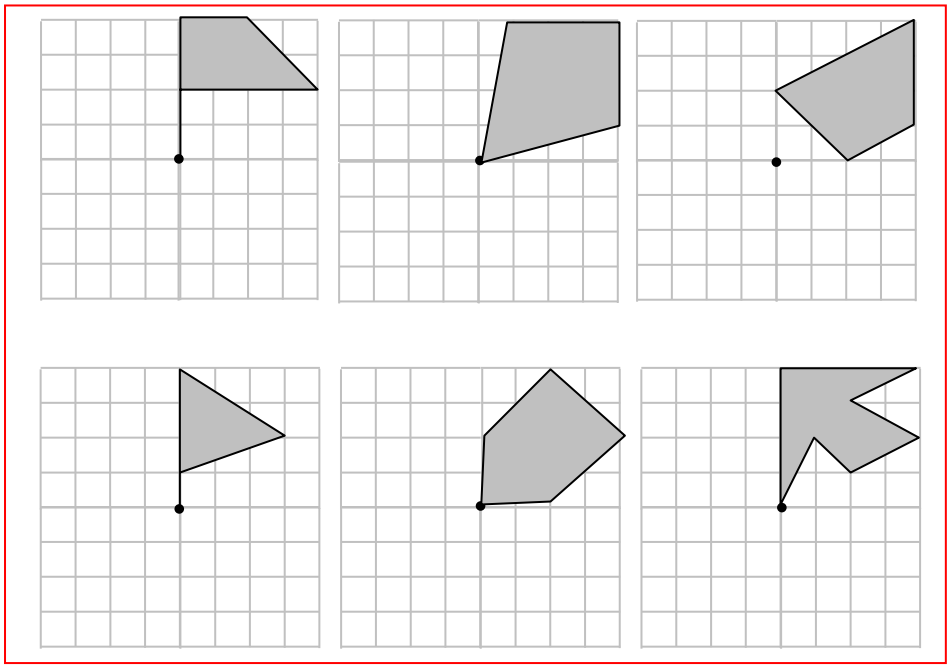
order of symmetry = 2



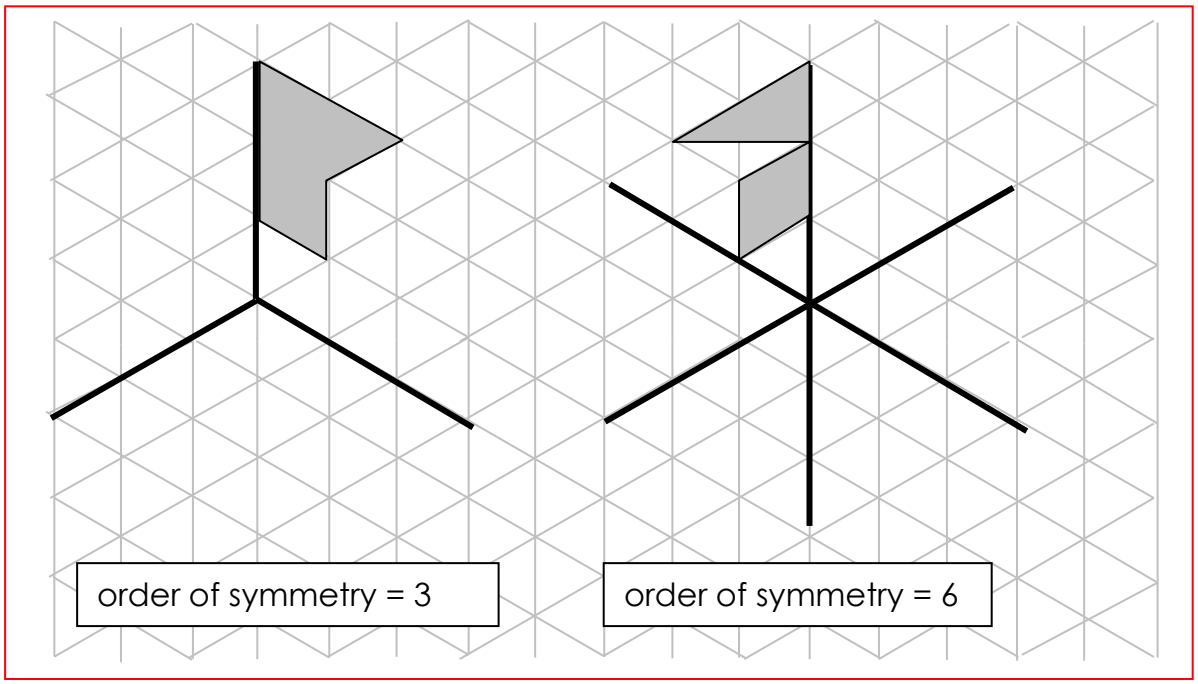
order of symmetry = 4



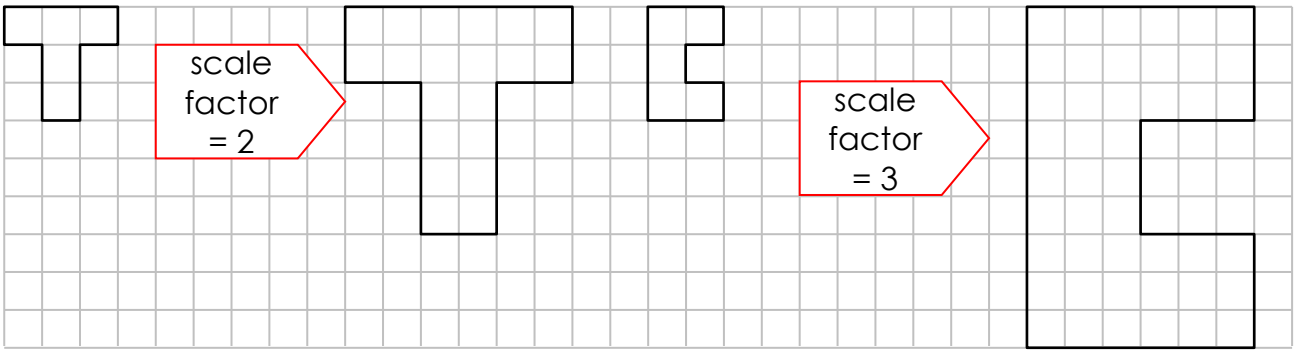
Exercise 5 Complete these patterns



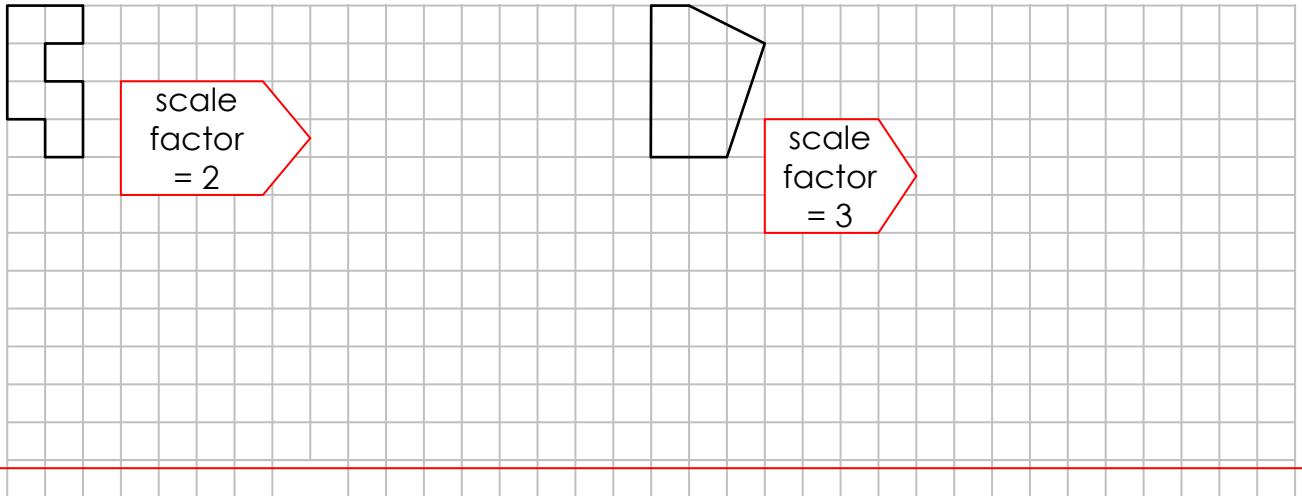
Exercise 6 Complete these patterns



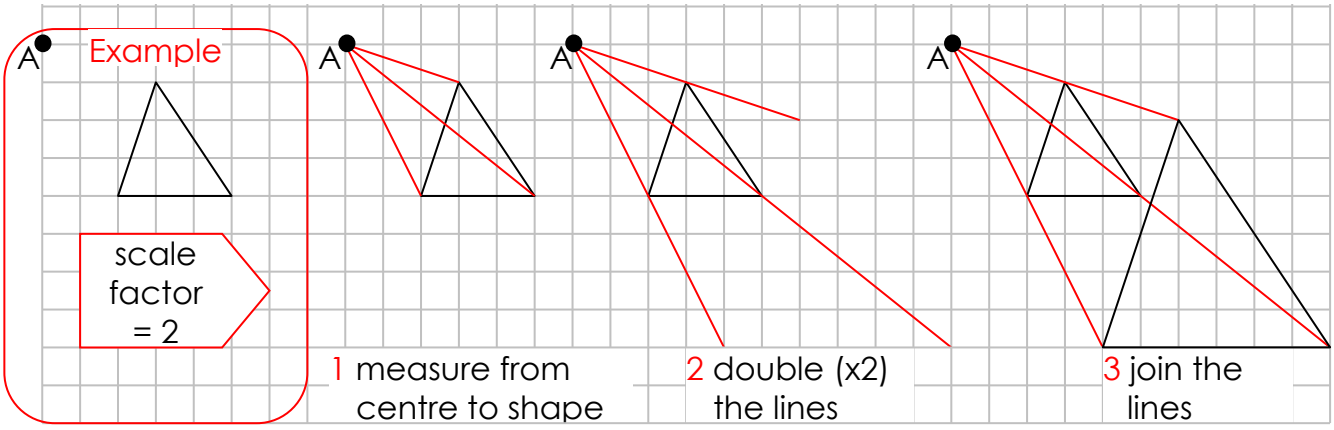
Enlargment



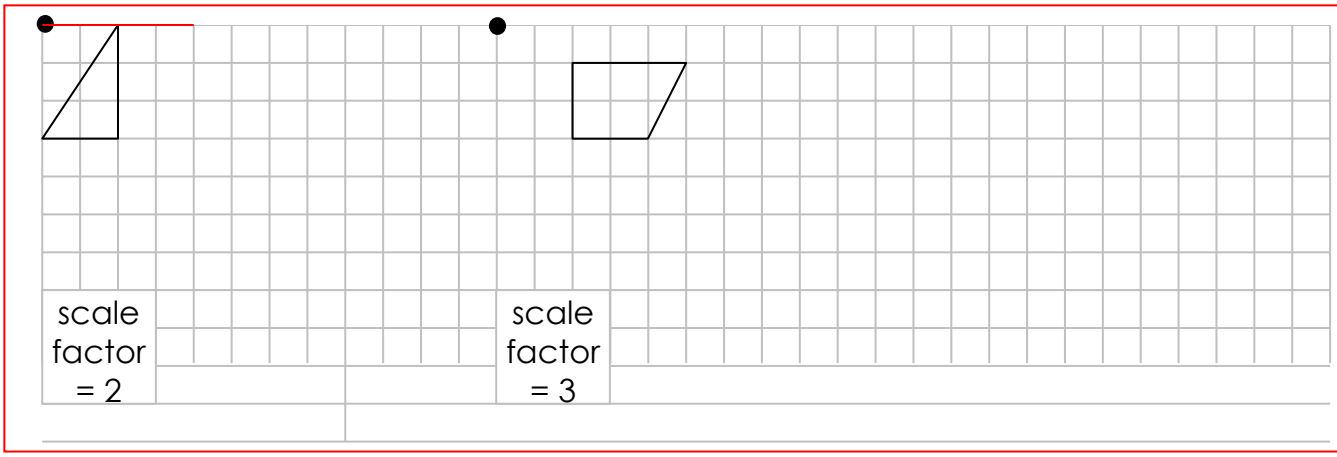
Exercise 7 **Enlarge these shapes**



Enlargement with a centre

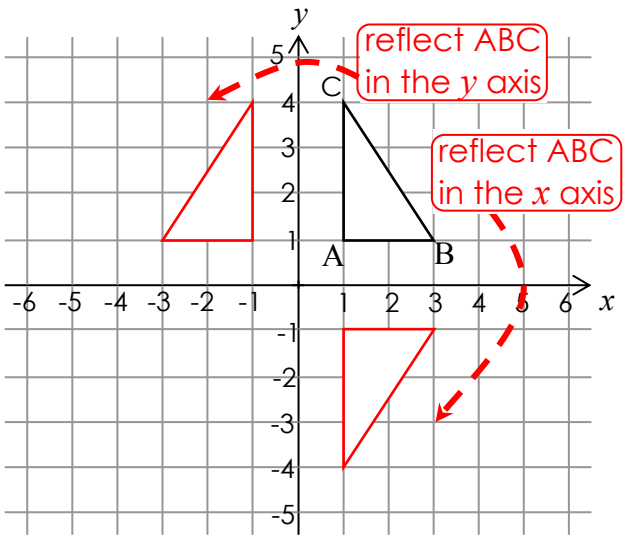


Exercise 8 **Enlarge these shapes**

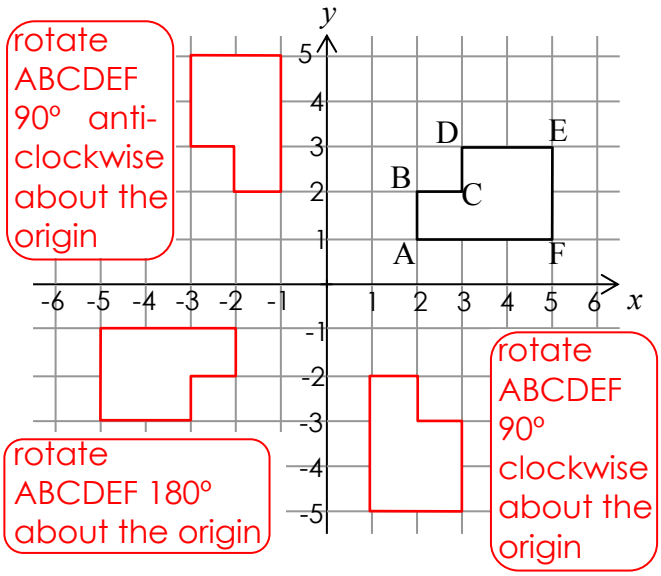


Reflection, rotation and coordinates

A(1,1) B(3,1) C(1,4)

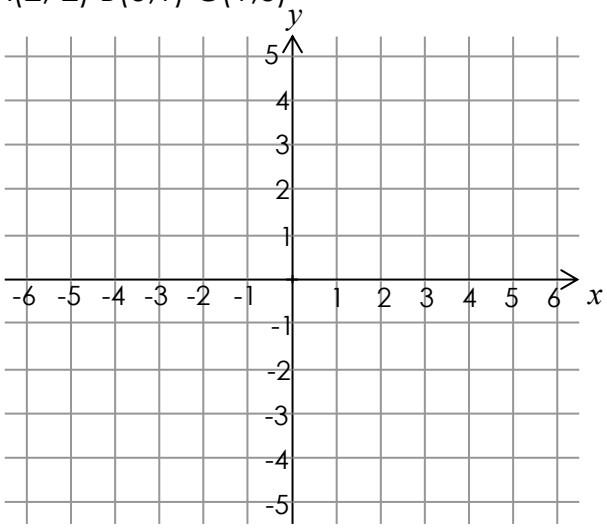


A(2,1) B(2,2) C(3,2) D(3,3) E(5,3) F(5,1)

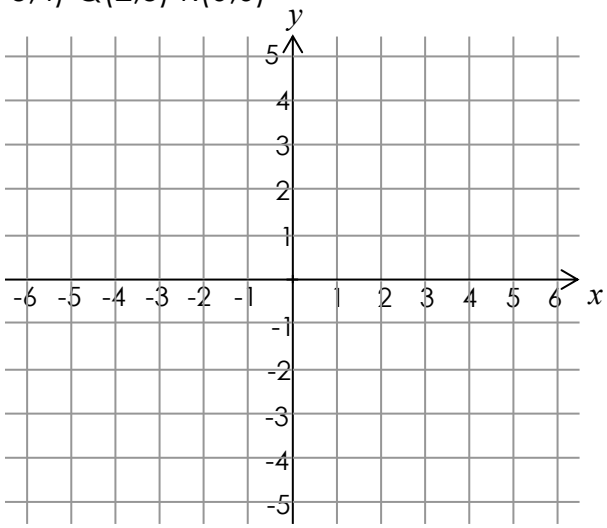


Exercise 9

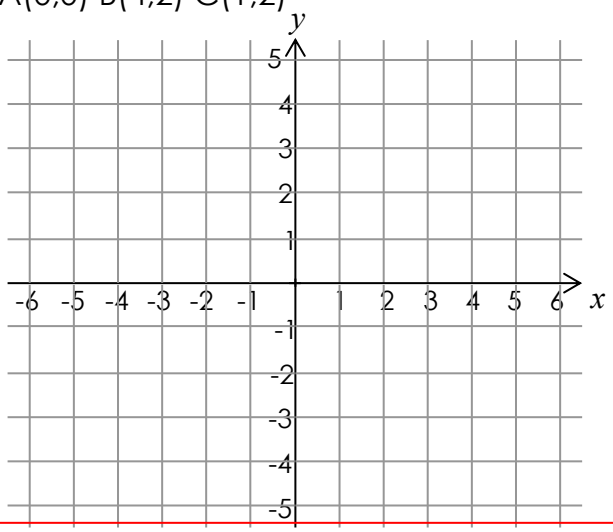
Reflect the triangle ABC in the y axis
A(2,-2) B(5,1) C(1,3)



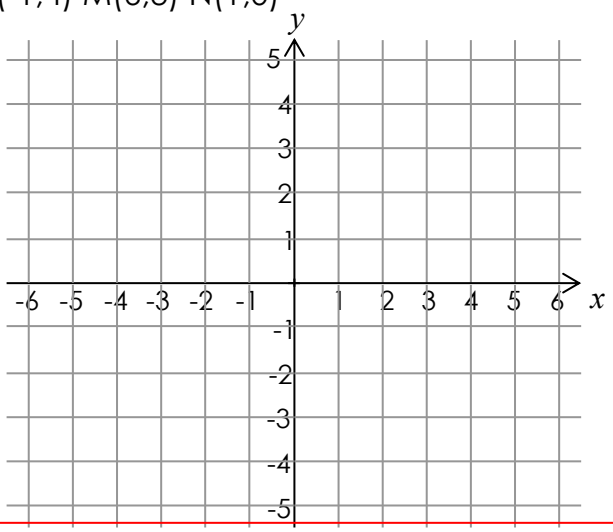
Reflect the triangle PQR in the x axis
P(-3,4) Q(2,3) R(0,0)



Rotate the triangle ABC 90° clockwise about the origin
A(0,0) B(4,2) C(1,2)

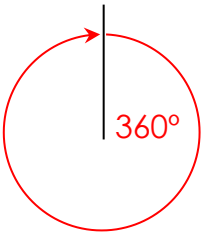


Rotate the triangle LMN 180° about the origin
L(-1,4) M(6,3) N(1,0)

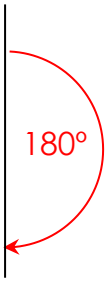


G7 Angle Relationships

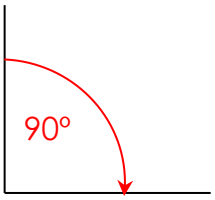
full turn



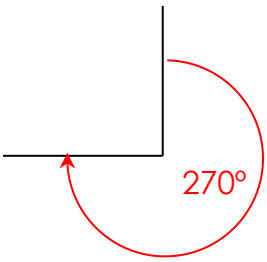
half (½) turn



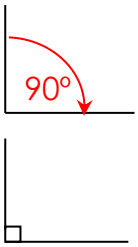
quarter (¼) turn



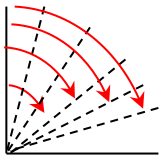
three quarter (¾) turn



right angle

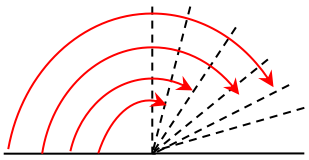


acute



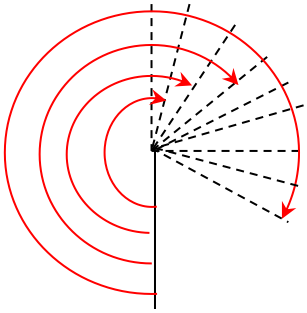
0 to 90°

obtuse



90° to 180°

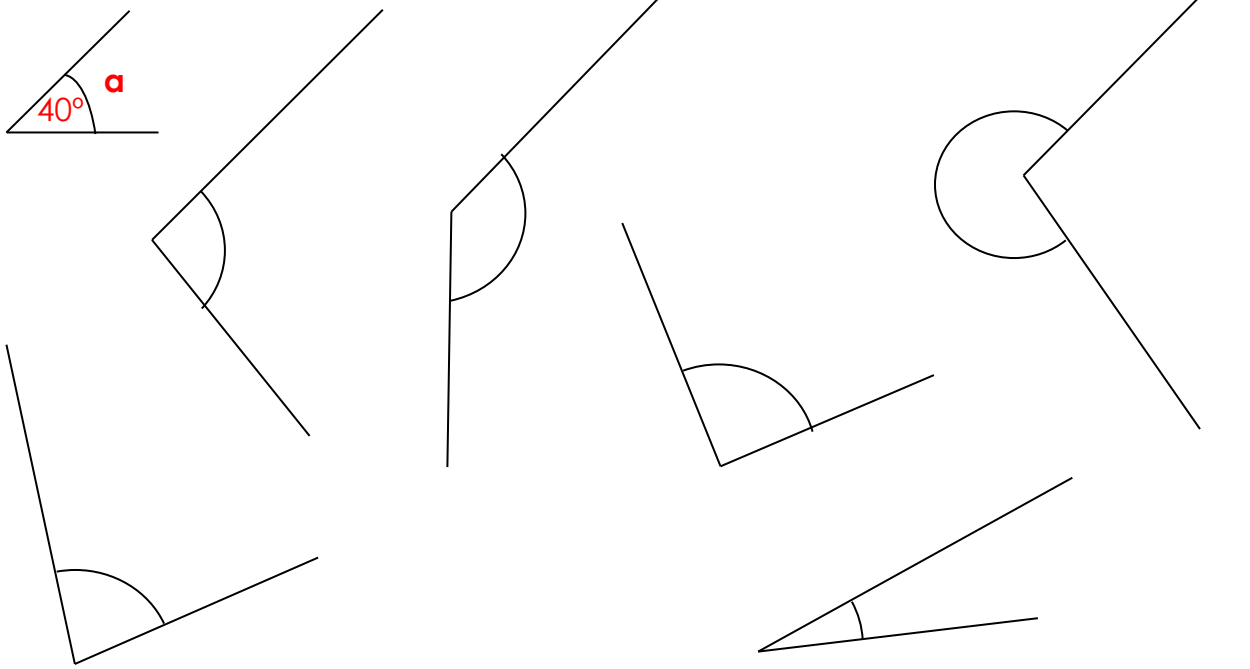
reflex



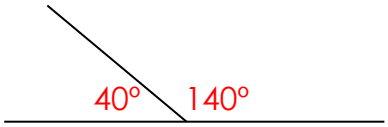
180° to 360°

Exercise 1 **Measure and mark these angles**

a = acute **b** = obtuse **r** = reflex

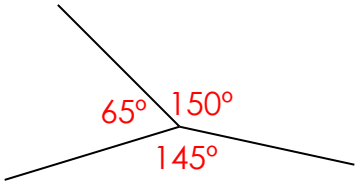


Adding to 180°



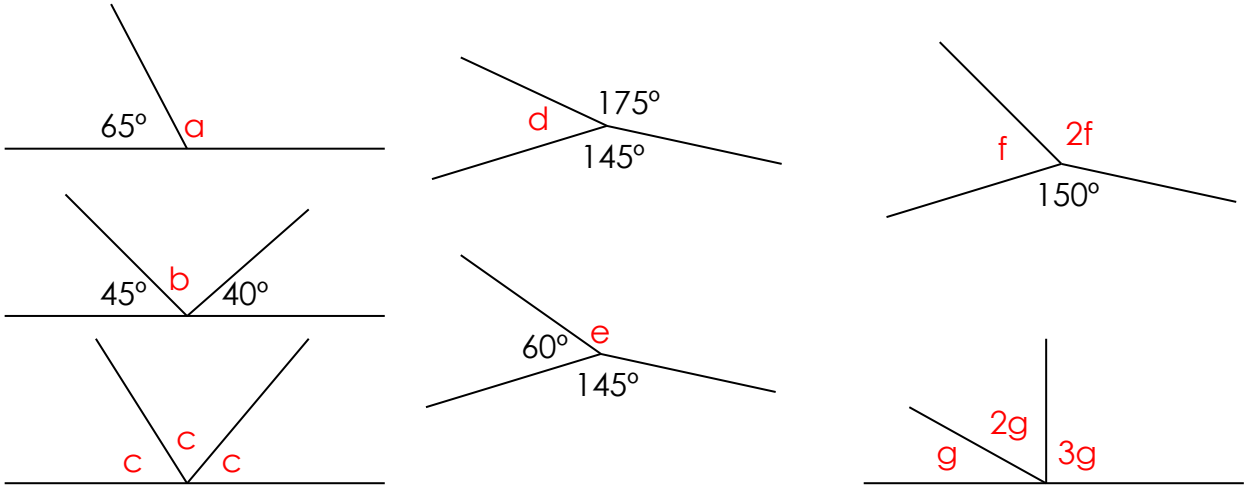
40 + 140 = 180° (half turn)

Adding to 360°



145 + 150 + 65 = 360° (full turn)

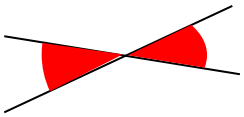
Exercise 2 *Calculate* these angles (DO NOT MEASURE)



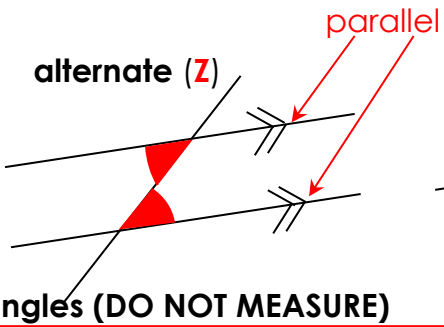
a = ___ b = ___ c = ___ d = ___ e = ___ f = ___ g = ___

Equal (=) angles

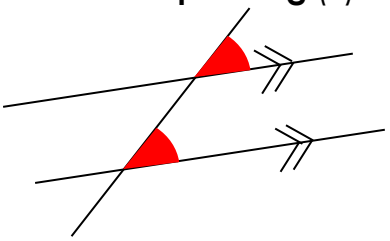
opposite (X)



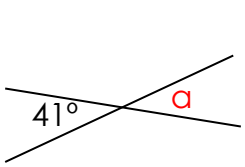
alternate (Z)

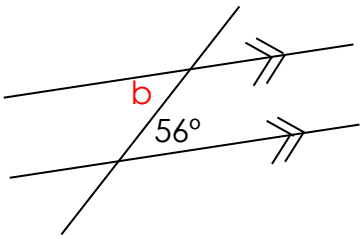


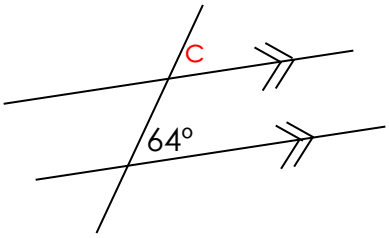
corresponding (F)

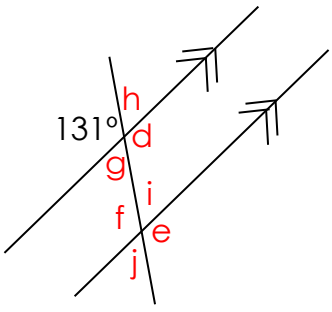


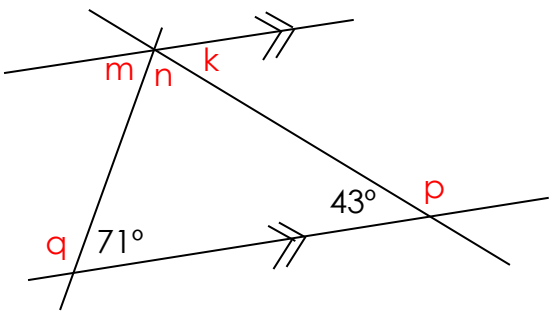
Exercise 3 Calculate these angles (DO NOT MEASURE)







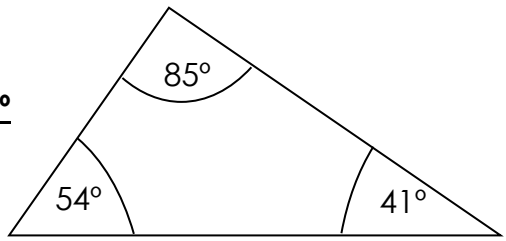




a =___ b =___
c =___ d =___
e =___ f =___
g =___ h =___
i =___ j =___
k =___ m=___
n =___ p =___
q =___

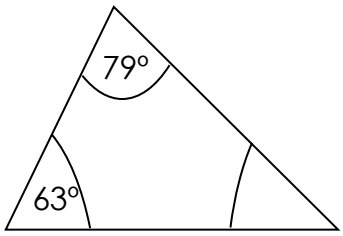
G8 Interior/Exterior Angles

Interior angles in a triangle add up to 180°

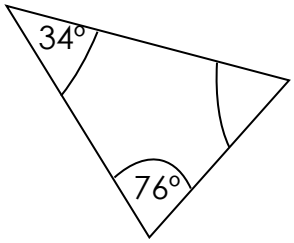


85 + 54 + 41 = 180°

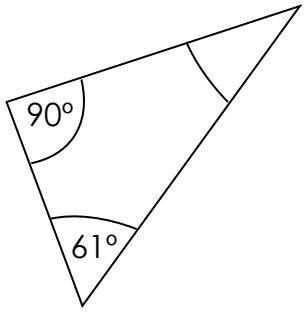
Exercise 1 **Work out the missing angles**



79 + 63 + _____ = 180°



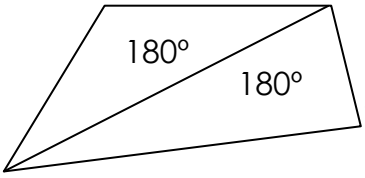
_____



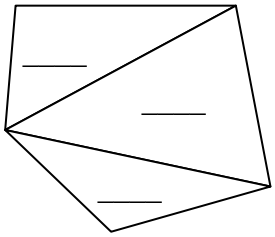
_____

Exercise 2 **Complete**

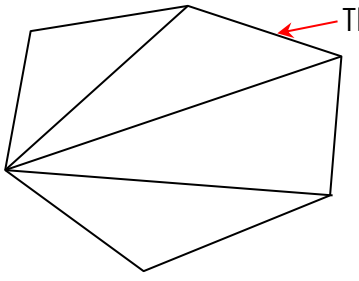
Interior angles of polygons



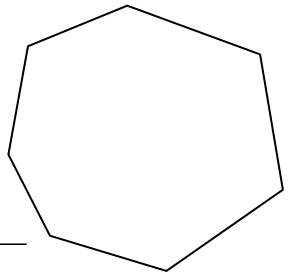
The interior angles of a **quadrilateral** add up to 360°



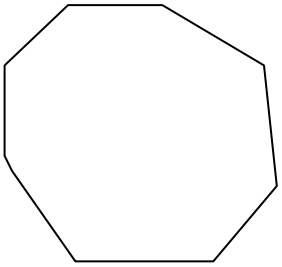
The interior angles of a **pentagon** add up to _____



The interior angles of a _____add up to _____

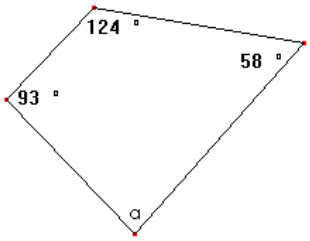


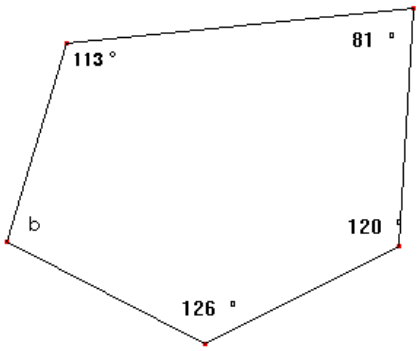
_____

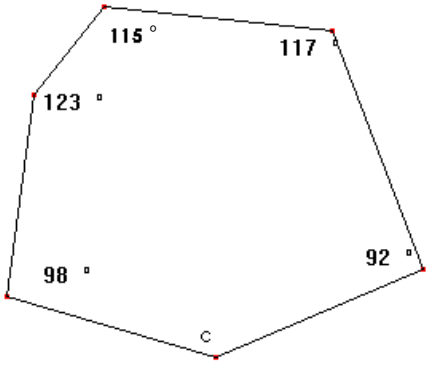


_____

Exercise 3 Complete







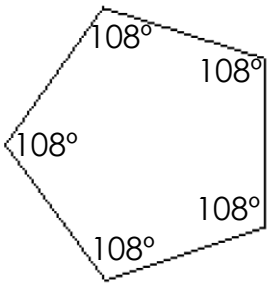
a = _____ b = _____ c = _____

Exercise 4

Angles in regular polygons

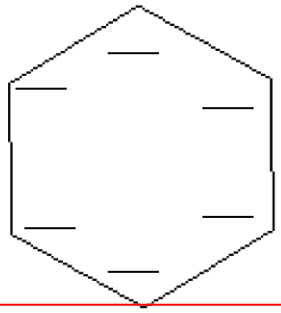
Pentagon

Interior angle
= 540 ÷ 5
= 108°

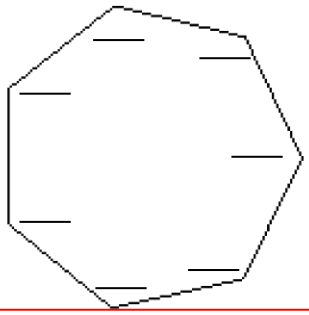


Hexagon

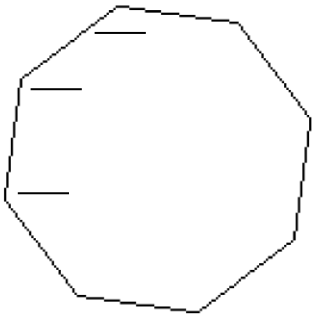
Interior angle
= _____ ÷ _____
= _____



_____
_____
= _____
=_____



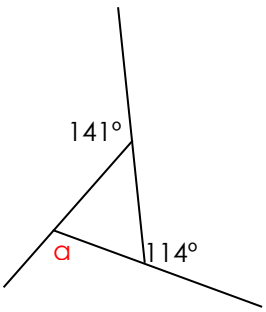
_____
_____
= _____
= _____

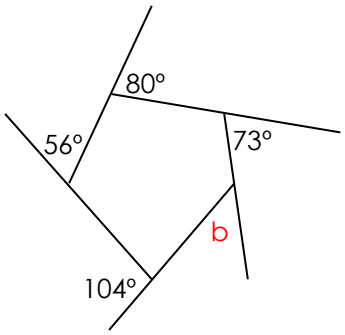


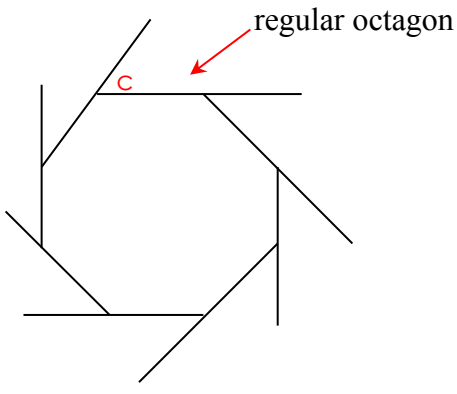
Exercise 5

Exterior angles

Exterior angles of all polygons add up to _____







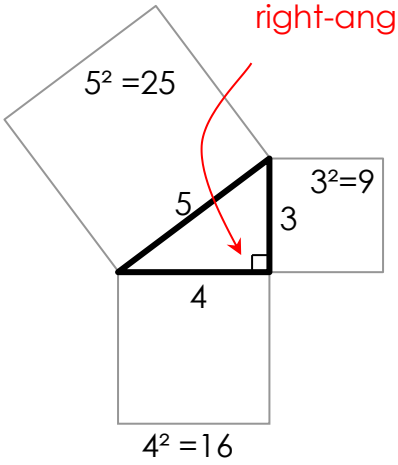
a = _____ b = _____ c = _____

G10 Pythagoras' Theorem and Trigonometry

The **sides** of this triangle are 3cm 4cm and 5cm

5cm is the **longest** side
It is called the **hypotenuse**

5^2 is five **squared** (5 x 5)



a = 3 cm
b = 4 cm
c = 5 cm

$a^2 = 9$ $b^2 = 16$ $c^2 = 25$

9 + 16 = 25
$3^2 + 4^2 = 5^2$
$a^2 + b^2 = c^2$

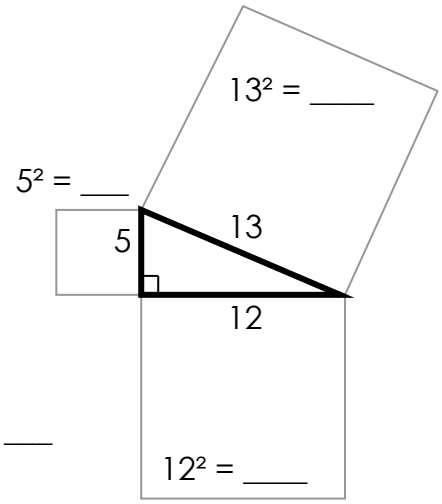
This is Pythagoras' Theorem

Exercise 1 Complete

a = 5
b = ___
c = ___

a^2 = 25
b^2 = ___
c^2 = ___

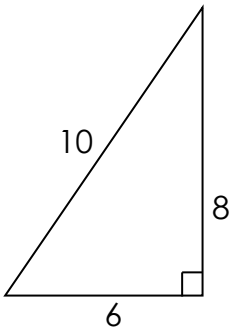
$a^2 + b^2 = c^2$
25 + ___ = ___



a = ___
b = ___
c = ___

a^2 = ___
b^2 = ___
c^2 = ___

$a^2 + b^2 = c^2$
___ + ___ = ___

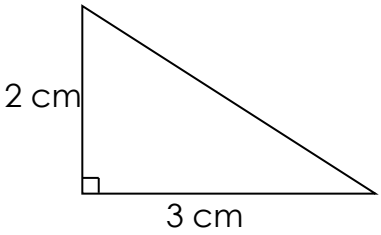


Finding missing sides

Use a calculator

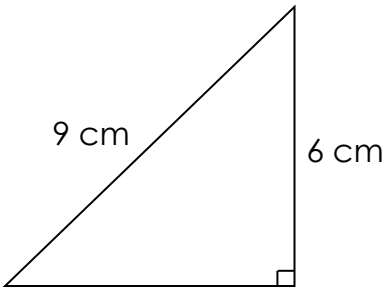
$a = 2$ $b = 3$
$a^2 = 4$ $b^2 = 9$

$a^2 + b^2 = c^2$
4 + 9 = 13
c^2 = 13
$c = \sqrt{13} =$ 3.61 cm



$a = 6$ $c = 9$
$a^2 = 36$ $c^2 = 81$

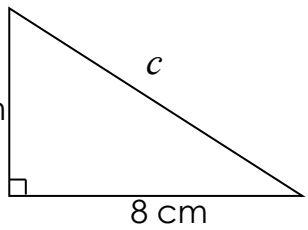
$a^2 + b^2 = c^2$
36 + b^2 = 81
b^2 = 45
$b = \sqrt{45} =$ 6.71 cm



Exercise 2 Complete

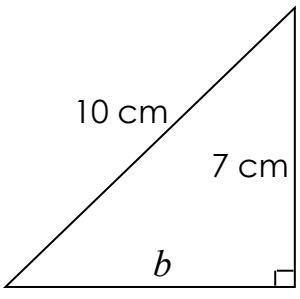
a = ___ b = ___
a^2 = ___ b^2 = ___

$a^2 + b^2 = c^2$
___ + ___ = ___
c^2 = ___
$c = \sqrt{\_\_\_} =$ ___ cm



a= ___ c = ___
a^2= ___ c^2 = ___

$a^2 + b^2 = c^2$
___ + b^2 = ___
b^2 = ___
$b = \sqrt{\_\_\_} =$ ___ cm



Exercise 3 Find the missing sides of these triangles

a= ___ b = ___

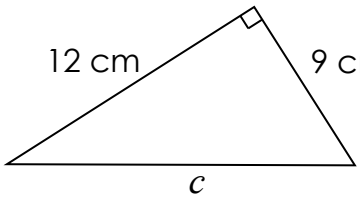
a^2=___ b^2 = ___

$a^2 + b^2 = c^2$

___ + ___ = ___

c^2 = ___

c = √___ = ____ cm



a= ____ c = ____

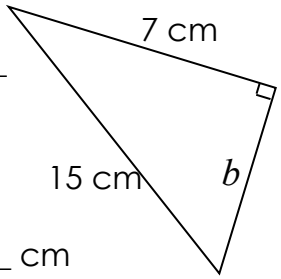
a^2=____ c^2 = ____

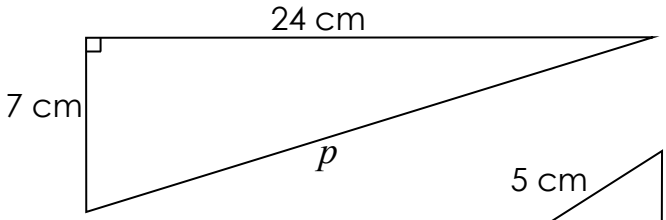
$a^2 + b^2 = c^2$

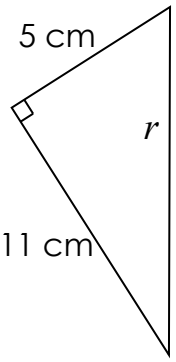
____+ b^2 = ____

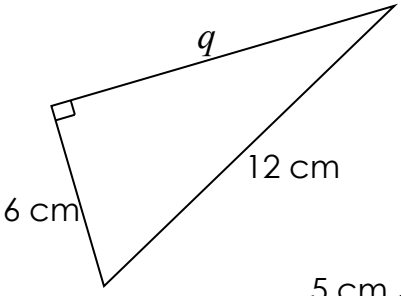
b^2 = ____

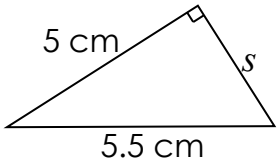
b = √____ = _____ cm



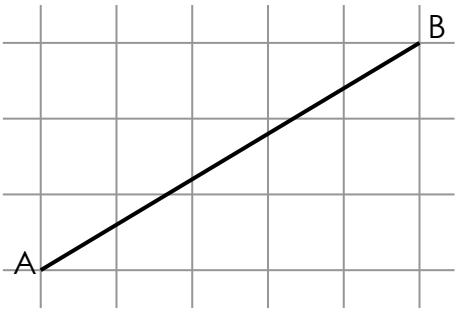




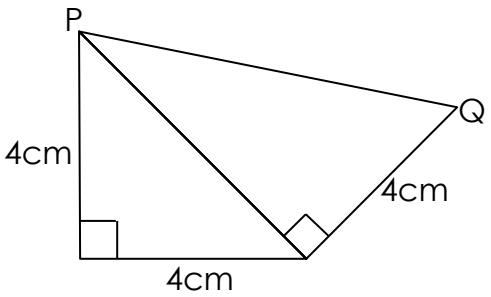




Exercise 4 Calculate the missing lengths



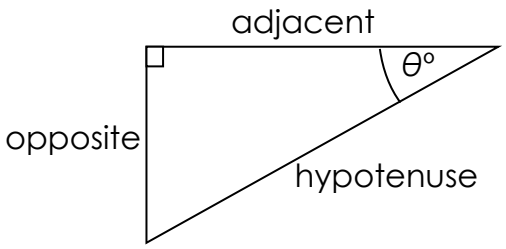
Find the length AB



Find the length PQ

Trigonometry

Right angled triangles

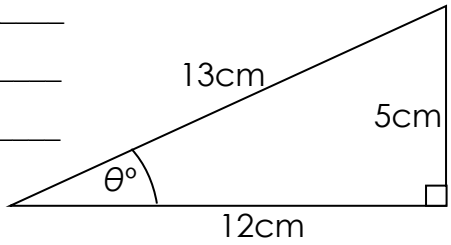


Exercise 1 **Complete**

hypotenuse = _____

opposite = _____

adjacent = _____

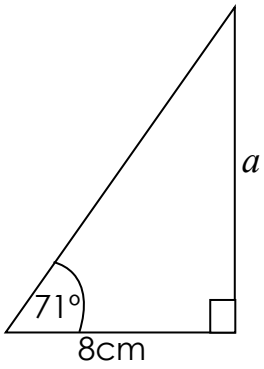


Using tan to find lengths

opposite = adjacent x **tan** θ
adjacent = opposite ÷ **tan** θ

Use a calculator

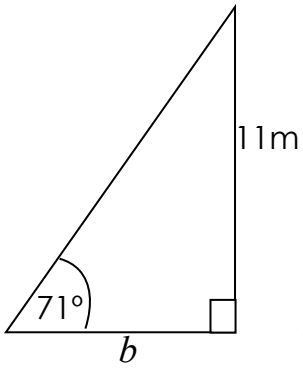
Exercise 5 **Complete**



opposite=adjacent x tan θ

a = 8 x tan 71

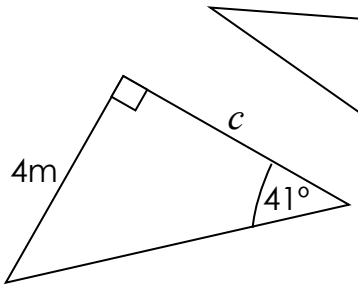
= 23.23cm



adjacent=opposite ÷ tan θ

b = ____ ÷ tan____

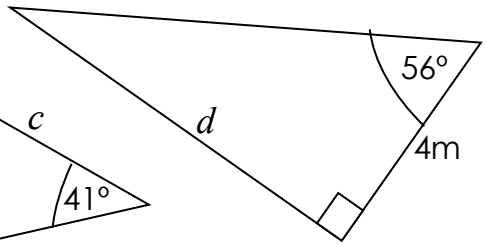
= _____cm



adjacent=opposite ÷ tan θ

c = ____ ÷ tan____

= _____cm



_____=_____

d = ____ x tan____

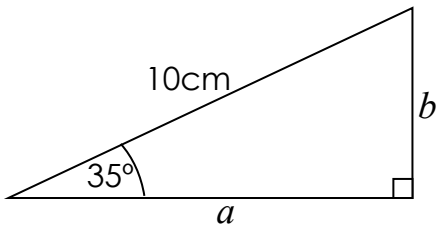
= _____cm

Using sin and cos to find lengths

hypotenuse x **sin** θ = opposite
opposite ÷ **sin** θ = hypotenuse

hypotenuse x **cos** θ = adjacent
adjacent ÷ **cos** θ = hypotenuse

Exercise 6 **Complete**



adjacent=hypotenuse x cos θ

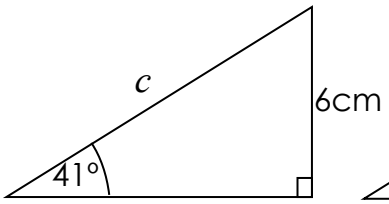
a = 10 x cos 35

= 8.19 cm

opposite=hypotenuse x sin θ

b = ____ x sin____

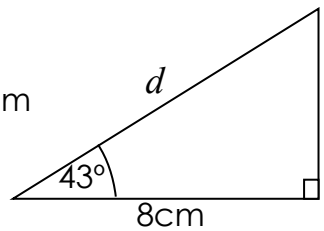
= _____cm



hypotenuse=opposite ÷ sin θ

c = ____ ÷ sin____

= _____cm

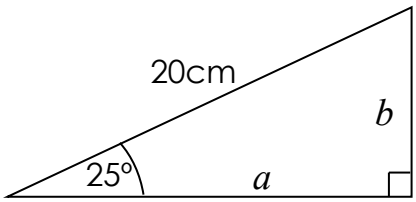


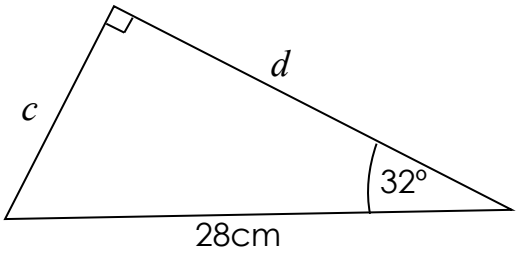
hypotenuse=adjacent ÷ cos θ

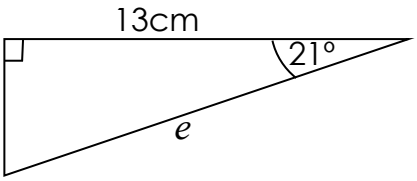
d = ____ ÷ cos____

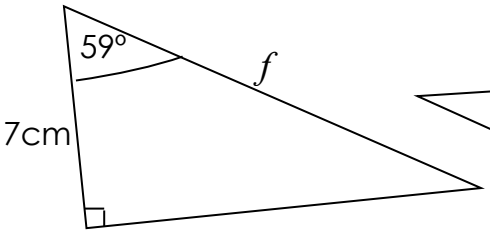
= _____cm

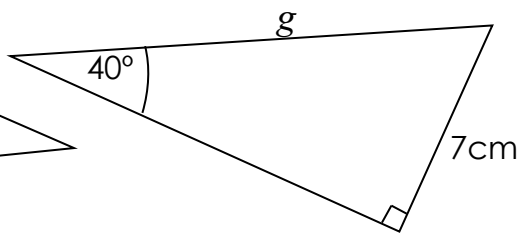
Exercise 7 **Use sin and cos to find the missing lengths**



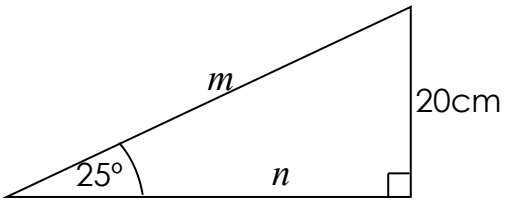


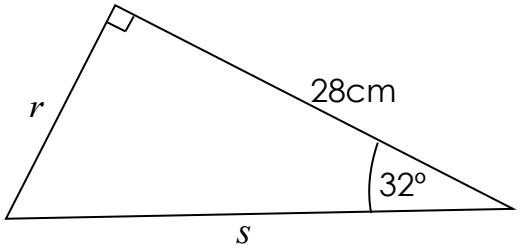


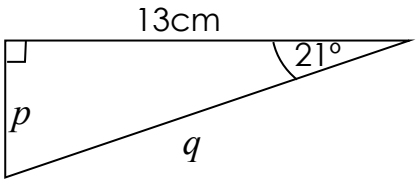


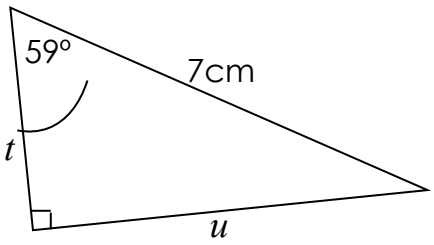


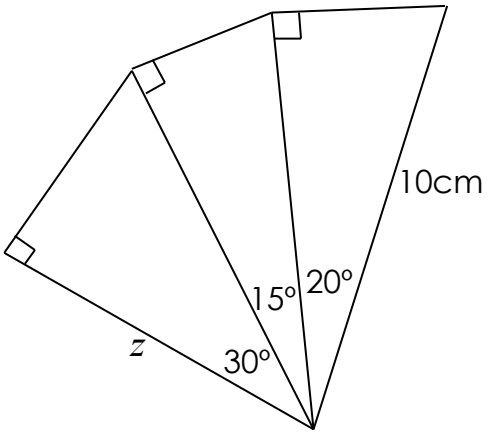
Exercise 8 **Use sin, cos and tan to find these lengths**

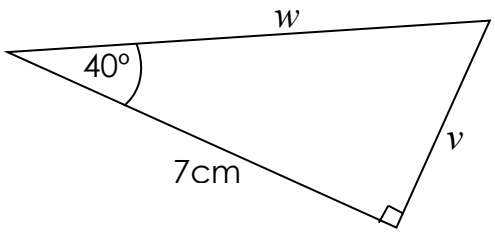




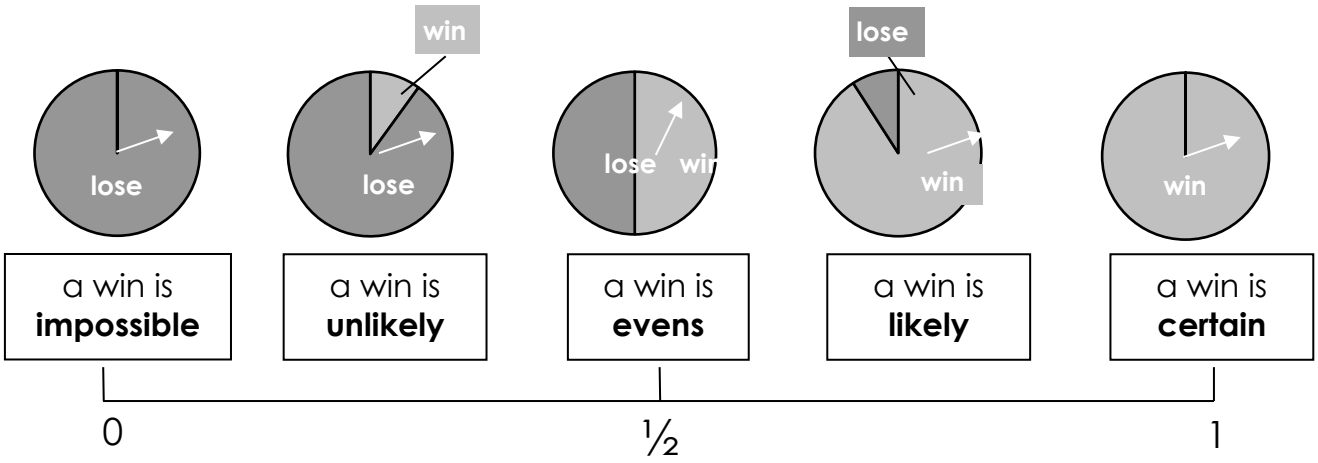




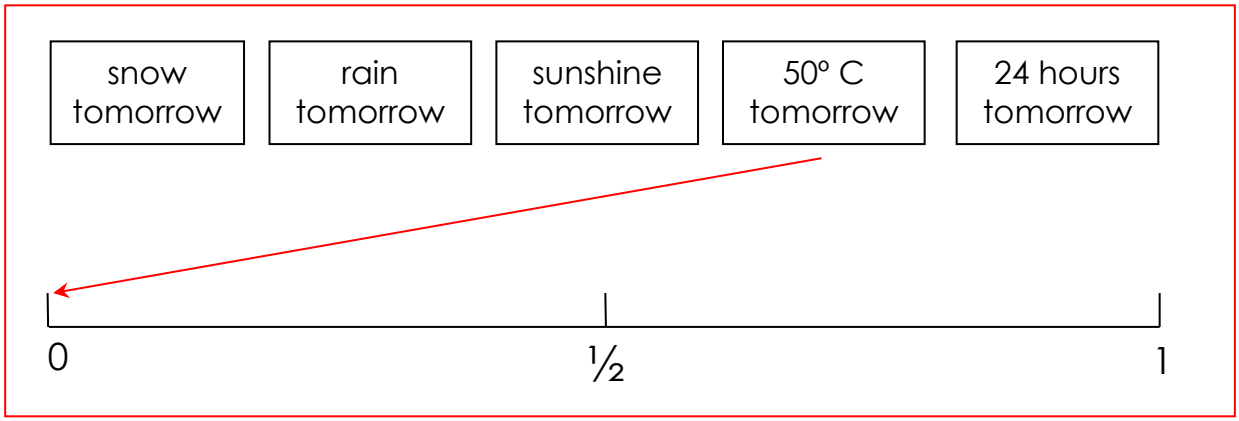




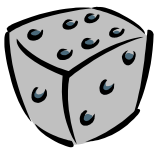
P1Probability Experiments

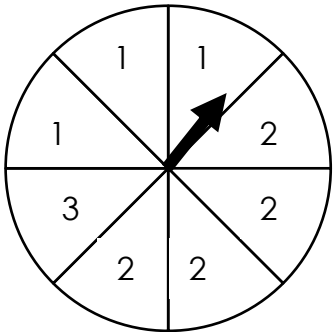


Exercise 1 **Draw arrows to complete**



Exercise 2 **Draw arrows to complete**

Roll a dice 	a 6	**impossible**
	a 7	**unlikely**
	more than 0 (> 0)	**evens**
	less than 6 (<6)	**likely**
	an odd number (1,3,5)	**certain**



Probability

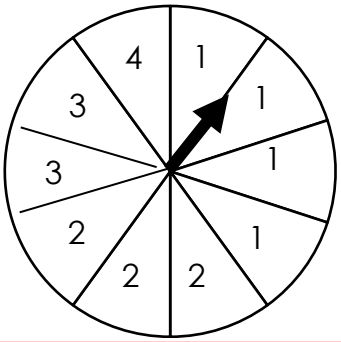
the probability of scoring 1 is $\frac{3}{8}$ P(1) = $\frac{3}{8}$

the probability of scoring 2 is $\frac{1}{2}$ P(2) = $\frac{1}{2}$

the probability of scoring 3 is $\frac{1}{8}$ P(3) = $\frac{1}{8}$

P2 Probabilities Add to One

Exercise 3 Complete



Probability

the probability of scoring 1 is ___ P(1) = ___

the probability of scoring 2 is ___ P(2) = ___

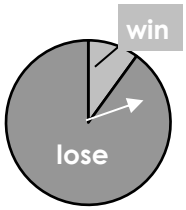
the probability of scoring 3 is ___ P(3) = ___

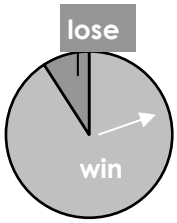
the probability of scoring 4 is ___ P(4) = ___

the probability of scoring 5 is ___ P(5) = ___

Exercise 4 Complete

Probabilities add to 1





Complete

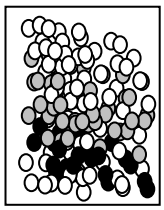
If P(win) = 10%
then P(lose) = 90%

If P(win) = 90%
then P(lose) = ____

If P(win) = 60%
then P(lose) = ____

If P(win) = 40%
then P(lose) = ____

Complete



P(white) = $\frac{8}{15}$

P(black) = $\frac{4}{15}$

P(grey) = —

P(white) = 0.7

P(black) = 0.1

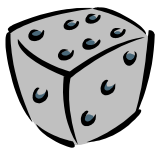
P(grey) = _____

P(white) = 55%

P(black) = 30%

P(grey) = _____

Probability of something NOT occurring



P(4) = $\frac{1}{6}$ so P (NOT a 4) = $\frac{5}{6}$

P(less than 3) = $\frac{2}{6}$ so P(NOT less than 3) = $\frac{4}{6}$

Exercise 5 Complete

1) P(winning) = 0.3 P(NOT winning) = _____

2) P(rain tomorrow) = $\frac{3}{8}$ P(NO rain tomorrow) = _____

3) P(England win) = $\frac{1}{4}$ P(England do NOT win) = _____

P3 Sample Spaces

Throwing two dice

Add the two scores:  +  = 9

Exercise 1 **Complete**

+	1	2	3	4	5	6
1	2	3				
2	3					
3						
4						
5						
6						

12 and 2 are the **least likely**

7 is the **most likely**

Adding two dice

The probability of getting 3 $P(3) = \frac{2}{36}$

The probability of getting 7 P(7) = ___

_____ P(12) = ___

The probability of getting **more than** 6 P(>6) = ___

The probability of getting **less than** 5 P(<5) = ___

The **difference** of the two scores:  -  = 1

Exercise 2 **Complete**

	1	2	3	4	5	6
1	0				4	5
2						
3				1		3
4		2		0		
5	4					
6	5					

_____ is the **least likely**

_____ is the **most likely**

The difference of two dice

The probability of getting 5 $P(5) = \frac{2}{36}$

The probability of getting 0 P(0) = ___

_____ P(3) = ___

The probability of getting **more than** 2 P(>2) = ___

The probability of getting **less than** 3 P(<3) = ___

Exercise 3 Complete

Tossing 2 Coins

Heads Heads Heads Tails Tails Heads Tails Tails

coin 1 \ coin 2	H	T
H	HH	HT
T	TH	TT

The probability of getting two heads P(HH) = $\frac{1}{4}$

The probability of getting two tails P(___) = ___

The probability of a head and a tail P(HT or TH) = ___

Throwing a dice and tossing a coin

	1	2	3	4	5	6
H	H1					
T				T4		

The probability of getting a head and a three P(H3) = ___

The probability of getting a tail and a four P(T4) = ___

The probability of getting a tail and any number P(T) = ___

These are the **suits** in a **pack** of cards Black ● Red ●

Spades (**S**) ♠ Hearts (**H**) ♥ Diamonds (**D**) ♦ Clubs (**C**) ♣

Exercise 4 Complete

Pick **two** cards from a pack

What's the probability of getting a spade and a club?

P(SC or CS) = ___

What's the probability of getting two of **the same** suit?

P(HH or SS or _____) = ___

What's the probability of getting **different** suits? ___

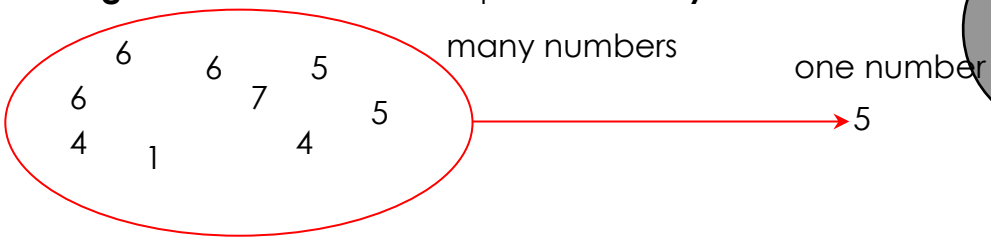
What's the probability of getting two **red** suits? ___

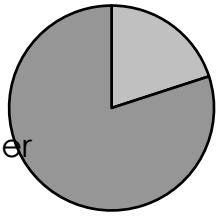
What's the probability of getting **different** coloured suits? ___

Card 1 \ Card 2	S	H	D	C
S				
H				
D				
C		CH		

S1 One Variable Statistics

An **average** is **one number** that represents **many numbers**





The **mode**: 1 4 4 5 5 6 6 6 7 there are **more** 6's so 6 is **the mode**

The **median**: 1 4 4 5 5 6 6 6 7 5 is in the **middle** so 5 is the **median**

smallest → biggest

add all the numbers

The **mean**: $\frac{1+4+4+5+5+6+6+6+7}{9} = \frac{44}{9} = 4.889$ so 4.889 is the **mean**

divide (÷) by 9 (there are 9 numbers)

Exercise 1

Find the **mode**, the **median** and the **mean** of these numbers: 0 4 5 5 1

The **mode**: ____ there are **more** ____ so ____ is **the mode**

____
____
____ ____

The **median**: ____ ____ ____ ____ ____ ____ is in the **middle** so _____ is the **median**

The **mean**: _____ = ____ = ____ so _____ is the **mean**

____ ____

The **median**: If there are two numbers in the middle, find their **mid-point**

e.g. 1, 1, 3, 3, 4, 7, 7, 7, 8, 10 5.5 is the **mid-point** of 4 and 7, so 5.5 is the **median**

(7+4)÷2

Exercise 2 **Find the mode, median and mean of these sets of numbers:**

1) 1 6 4 8 3 5 2 2 2 1

2) 32 33 30 35 36 36 36 30

3) 10 14 16 13 10 10

4) 5 5 5 5 5 5 5 45

Data in Tables

how many

age	frequency
4	1
5	3
6	4
7	2
total	10

4
5 5 5
6 6 6 6
7 7

Calculating the mean

4	=4	4 x 1 = 4
5+5+5	=15	5 x 3 = 15
6+6+6+6	=24	6 x 4 = 24
7+7	=14	7 x 2 = 14
total	=57	total = 57

mode = 6 **mean** = 57 ÷ 10 = 5.7

median: the **mid-point** is 6 so the **median** is 6

Exercise 3 **Find the mode, median and mean of these data sets:**

age	frequency
6	2
7	10
8	7
9	1
total	_____

Calculating the mean
6 x 2 = 12

total =

mode = ___ **mean** = _____ = _____

median: the **mid-point** is _____ so the **median** is _____

brothers	frequency
0	14
1	19
2	10
3	5
4	2
total	_____

Calculating the mean

total =

mode = ____ **mean** = _____ = _____ **median** = _____

height	frequency
1<h≤1.2	3
1.2<h≤1.4	8
1.4<h≤1.6	12
1.6<h≤1.8	10
1.8<h≤2.0	7
total	_____

mid-points
1.1
1.3

Calculating the mean
1.1 x 3 = 3.3

total =

mode = ____<h≤____ **mean** = _____ = _____ **median** = ____<h≤____

Measuring spread

3, 5, 2, 7, 5, 4, 6, 3, 5 — All these numbers are close together
The **spread** is small

the biggest – the smallest = **the range**
range = 7 – 2 = **5**

3, 78, 45, 128, 5, 67, 53 — The **spread** is large

The **range** = 128 – 3 = **125**

Exercise 4

The range

1) 4, 8, 6, 7, 123, 1001, 1, 5, 1 — The **spread** is _____

The **range** = _____

2) 45, 47, 48, 48, 51, 46, 43 — The **spread** is _____

The **range** = _____

3) A small range is **consistent** and a big range is **inconsistent**

Which of these sets is the most **consistent**?

Set A	*Set B*	*Set C*
15, 6, 71, 34, 45, 56	15, 16, 18, 14, 16	26, 27, 29, 25, 20
range = _____	range = _____	range = _____

Set ___ is the **most** consistent

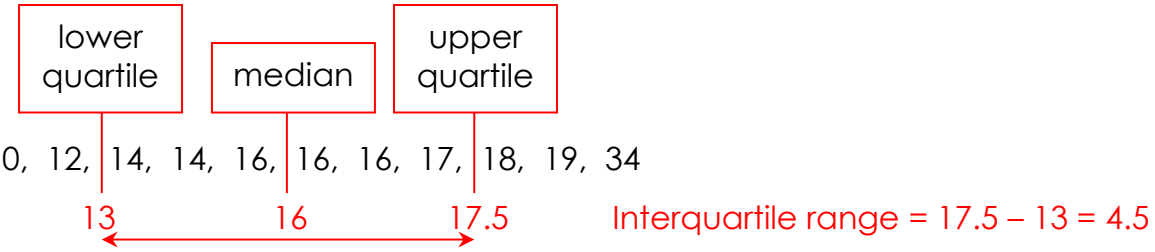
Set ___ is the most **inconsistent**

Quartiles

Quartiles measure spread, but ignore the **very small** and the **very big** numbers

14, 16, 1, 17, 18, 12, 14, 16, 16, 19, 10, 34

Put the numbers in order and divide into four groups:



The **interquartile range** is a measure of **spread**

Exercise 5 **Find the interquartile range:**

1) 34, 36, 3, 37, 48, 22, 14, 36, 16, 159, 31, 34

Put the numbers in order and divide into four groups:

3, 14, 16, 22, 31, 34, 34, 36, 36, 37, 48, 159

Interquartile range = _____

2) Find the **interquartile range** and the **median** of these sets of data:

a) 1, 5, 3, 6, 4, 8, 6, 39

b) 56, 57, 54, 42, 57, 58, 10, 100, 49, 51, 53, 56, 45, 25, 43, 43

c) 71, 75, 73, 66, 46, 78, 76, 19

S2 Statistical Charts

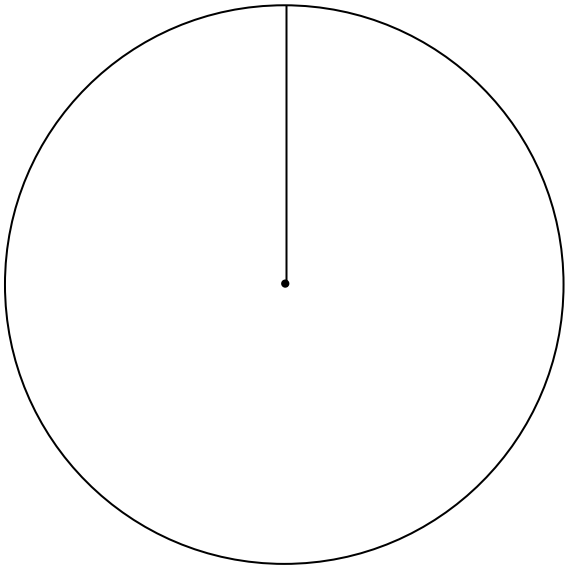
Pie Charts

Exercise 1 **Complete**

colour	frequency	angle
red	1	x 36 = ____
blue	4	x 36 = ____
yellow	2	x 36 = ____
green	3	x 36 = ____

total = 10

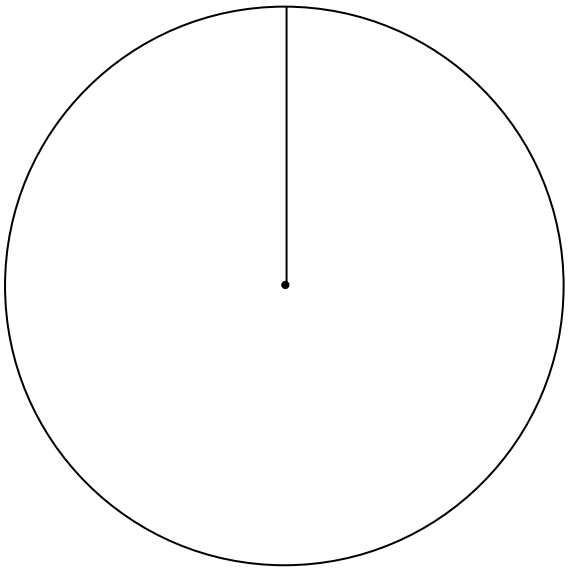
360 ÷ 10 = 36



food	proportion	angle
meat	12%	x ___= ____
fruit	18%	x ___= ____
vegetable	42%	x ___= ____
fish	9%	x ___= ____
dairy food	____%	x ___= ____

total = _____

360 ÷ _____= _____



Complete a pie chart for this data

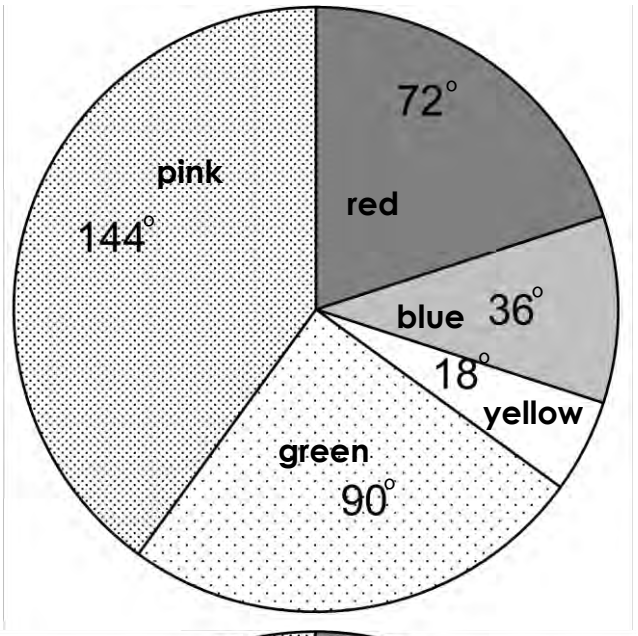
colour	fraction	angle
red	$^3/_8$	x ___= ____
blue	$^1/_8$	x ___= ____
yellow	$^1/_3$	x ___= ____
green	$^1/_6$	x ___= ____

Exercise 2 **Fill in the tables**

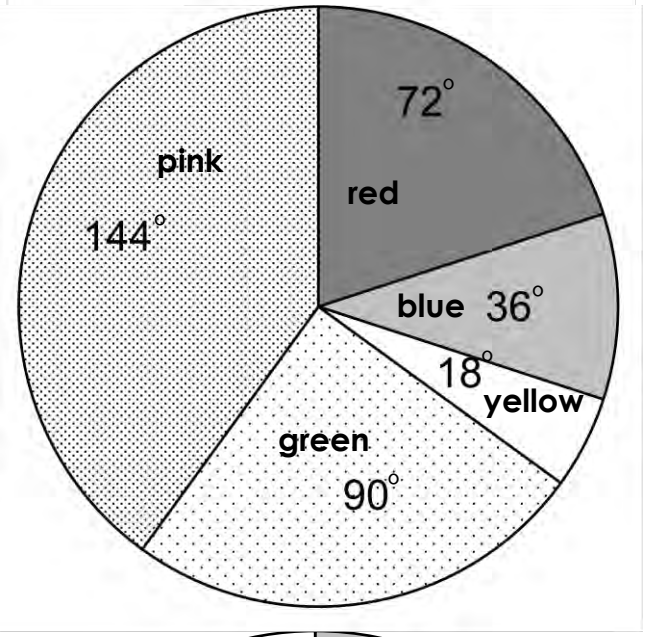
colour	frequency	angle
red	12	
blue		
yellow		
green		
pink		

What is the mode of this data?

_____



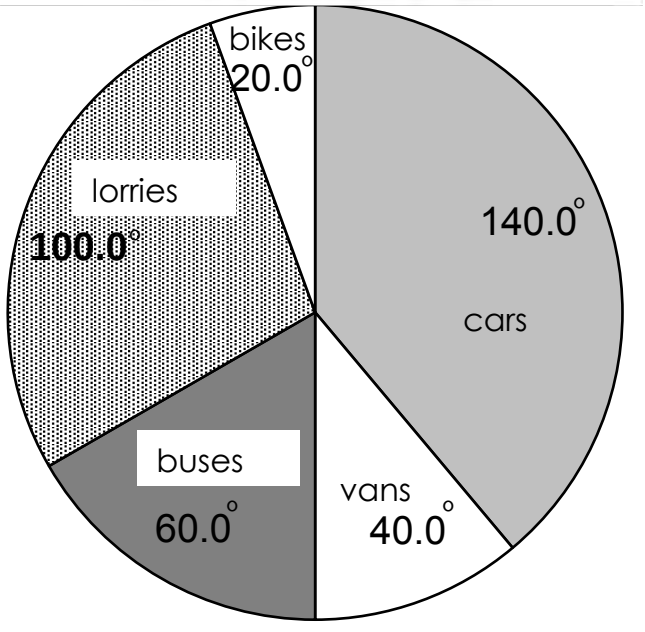
colour	frequency	angle
red	___%	
blue	___%	
yellow	___%	
green	___%	
pink	___%	



transport	**fraction**	angle
cars		
lorries		
vans		
buses		
bikes		

What is the mode of this data?

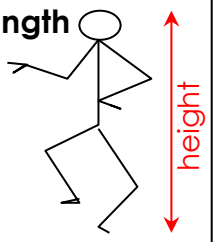
_____



S3 Scatter Diagrams

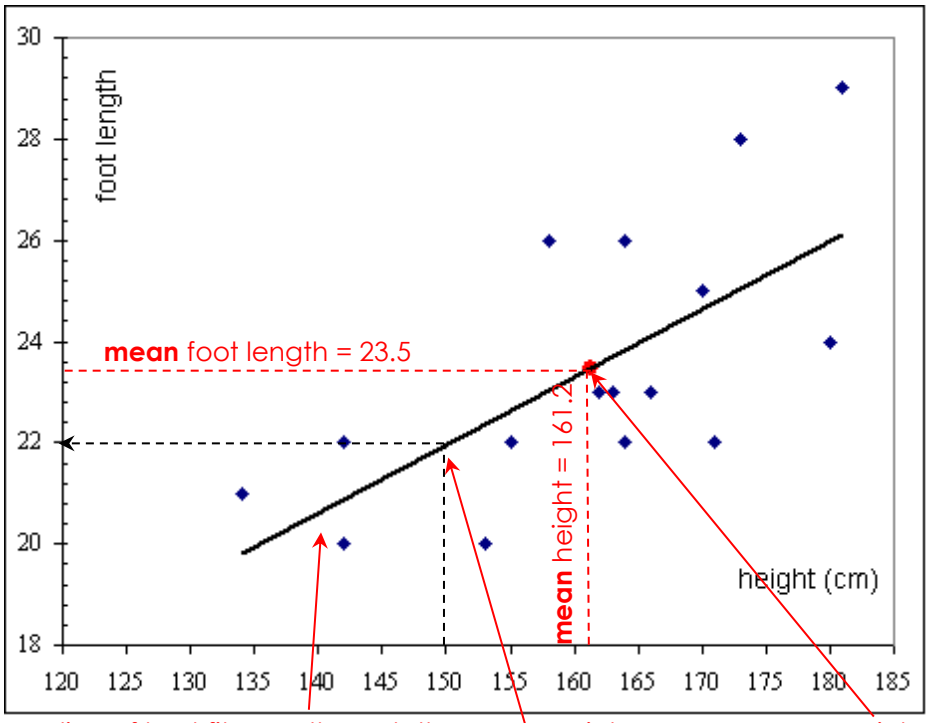
Scatter diagrams **compare** two **variables**

Example: This diagram compares **children's height** with their **foot length**



Most **short** people have **small** feet.
Most **tall** people have **big** feet.
This is **positive correlation**.

Height (cm)	Foot length (cm)
164	22
158	26
162	23
166	23
164	26
155	22
163	23
163	23
153	20
181	29
134	21
142	22
142	20
180	24
171	22
173	28
170	25
mean height =161.2	mean foot length =23.5



line of best fit goes through the mean point

mean point

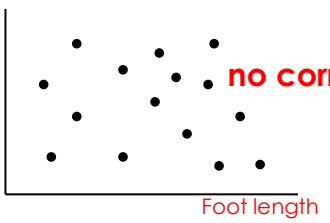
Example: If height = 150cm we **expect** foot length = 22cm

Temperature on a mountain

high up the mountain– **low** temperature
low down the mountain – **high** temperature

negative correlation

Comparing foot length and maths test score
no relationship



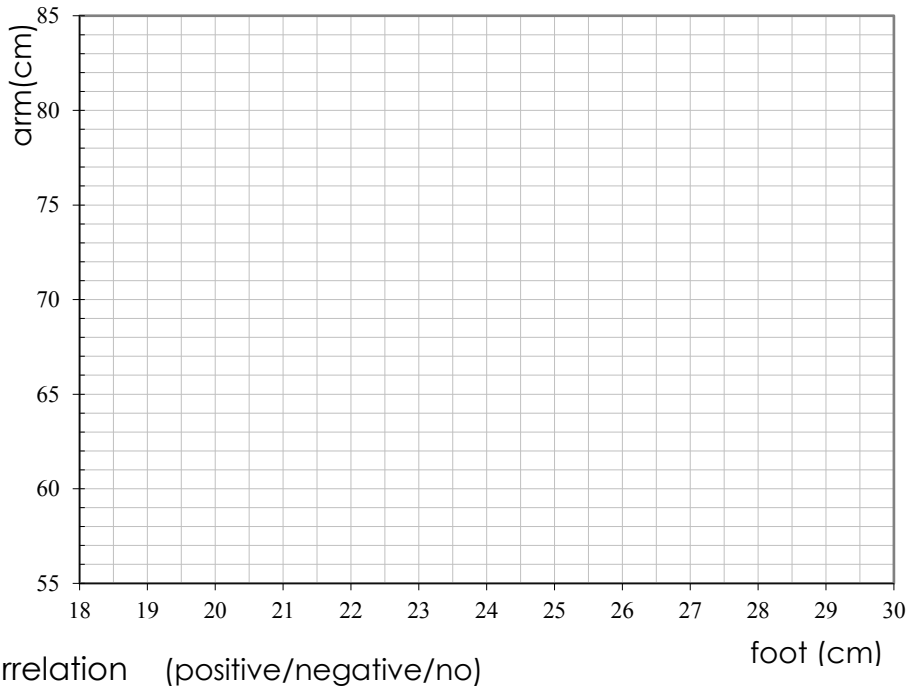
no correlation

To draw a Scatter diagram:
plot the points – plot the **mean point** – draw **line of best fit** – name the **correlation**

Exercise 1

1) Complete the table and plot the scatter diagram. Draw the **line of best fit**.

Foot (cm)	Arm (cm)
22	69
26	71
23	66
23	67
26	74
22	73
23	69
23	68
20	59
29	80
21	72
22	71
20	63
24	73
22	65
mean =	mean =



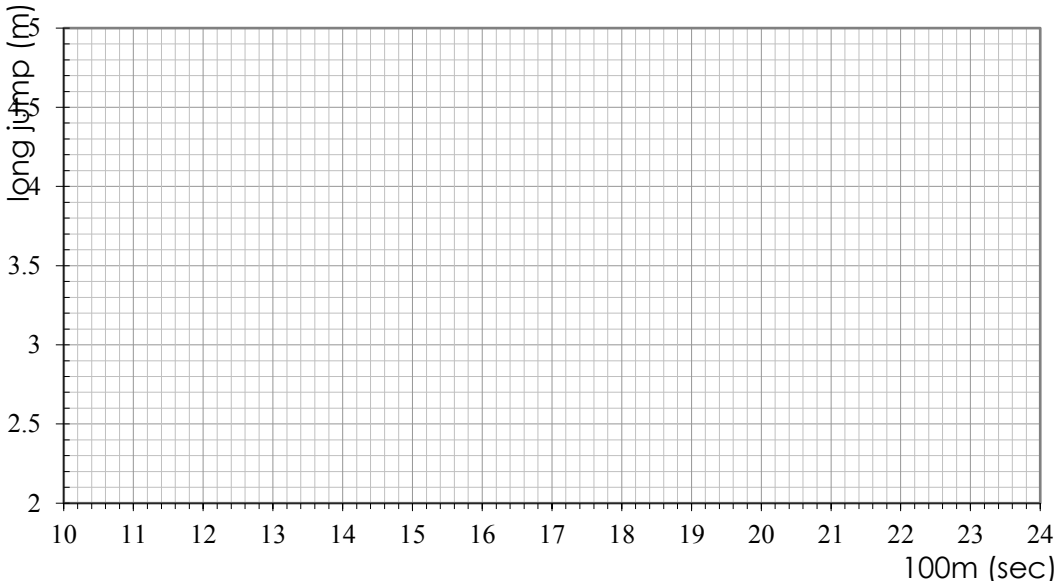
There is _____ correlation (positive/negative/no)

If foot length = 22cm we expect arm length = _____

If arm length = 75cm we expect foot length = _____

2) Complete the table and plot the scatter diagram. Draw the **line of best fit**.

100m (sec)	Long jump (m)
17.09	2.81
13.88	3.10
14.6	3.70
17.48	2.80
17.02	2.85
20.19	2.48
15.95	3.50
15.54	3.78
14.39	3.75
22.51	2.48
16.82	2.41
15.76	3.10
16.22	2.97
20.2	2.20
mean =	mean =



There is _____ correlation (positive/negative/no)

If the time is 15 seconds for the 100m, we expect the long jump = _____

If the long jump = 2.6m the expected 100m time = _____

Data-Chart-Analysis Answers

D6	C1	A2
D4	C2	A1
D5	C3	A6
D1	C4	A5
D3	C5	A4
D2	C6	A3

Source with acknowledgement:
https://www.channel4.com/news/factcheck/climate-change-in-ten-graphs