

# COUNT ON US

## SECONDARY CHALLENGE



## STUDENT WORKBOOK

**GET ENGAGED IN MATHS!**



## INTRODUCTION

The **Count on Us Secondary Challenge** is a maths tournament involving over 4000 young people from across London, delivered by the Mayor's Fund for London in partnership with the Jack Petchey Foundation. We hope that by taking part this year, you will become more confident in maths, you will develop your problem solving skills and you will boost your maths skills too.

The **Secondary Challenge** is made up of three rounds in different areas of maths. In the Spring Term, your school will select a team of 5 people to represent you in the Regional Heats. Before then, we hope that everyone should get the chance to work on these activities. We hope you will find them really good fun and you'll want to practice lots to get really good at them!

This book will explain how all the activities work and give you everything you need to try them out, practice them and get really good at puzzling, problem solving and fast paced number skills. You will work on geometry, algebra and numbers (including fractions and decimals), so everything that you do will help you with your ordinary maths lessons too.

The national curriculum in maths expects you to get really good at three things:

- (i) Fluency: you can do maths quickly and accurately, mostly in your head (NO reaching for a calculator when you see simple numbers!)
- (ii) Reasoning: you can see how things work mathematically and describe it
- (iii) Problem Solving: you can find a mathematical way to solve problems.

The three rounds of the challenge will help you work on these skills in different areas of maths.

ROUND	ACTIVITY	FOCUS	CONTENT
1	<b>The Game of Hex: strategy Geometric puzzling: Gridlines Geometry Card Game</b>	Fluency Problem solving	Geometry
2	<b>Whole number, Fractions &amp; Decimals, Algebra &amp; Exponents challenge using the 24® Game.</b>	Fluency	Number
3	<b>Story based problem solving activity using algebra and coding skills.</b>	Fluency Problem solving Reasoning	Algebra

## ROUND 1: GEOMETRIC GAMES AND PROBLEMS

Professional mathematicians explore mathematics having no idea what the outcome might be. This needs them to be prepared to carry on even when they have no idea at all. They never give up!

The English mathematician Andrew Wiles describes what this feels like in a BBC Horizon programme 'Fermat's Last Theorem', which can easily be found with an internet search. Just watch the first two minutes and you'll be hooked!

This round is in two parts:

1. Playing the game of Hex with your team as the counters!
2. Solving geometric problems with the Gridlines: Geometry game.

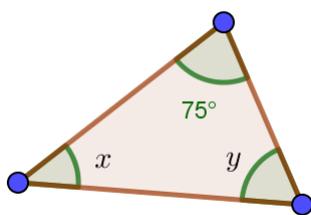
**Use the reference to find out how the game of Hex works and get plenty of practice. You can find many videos showing Hex strategy on YouTube. When you are ready, play Hex in teams on a floor mat. It is very different being one 'piece' in a team compared to playing on your own!**

The Game of Hex: <https://www.maths.ed.ac.uk/~csangwin/hex/index.html>

**The GridLines: Geometry game has lots of geometric situations in; (i) Angle relationships, (ii) Using Pythagoras' theorem, (ii) Finding areas.**

In the game you must find particular solutions to general situations, using some of the ten number cards you will be allowed to use.

For example, here is a situation:



You know that  $x + y = 180 - 75 = 105^\circ$

So, you would try to make for two numbers that fit.

E.g.  $50^\circ$  and  $55^\circ$ , or  $20^\circ$  and  $85^\circ$ , or  $37^\circ$  and  $68^\circ$  etc.

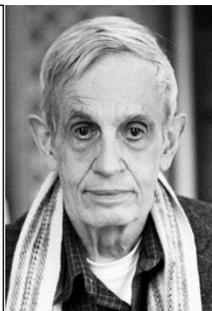
If you cannot find numbers to fit, there are always two other problems to work on.

To prepare for this round, you should get lots of practice solving standard geometry problems in the three areas. Then practice with the GridLines: Geometry game set.

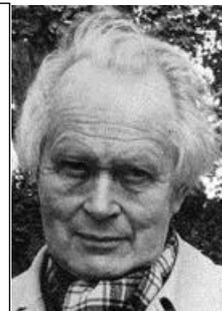
## THE GAME OF HEX

Hex is a strategy game. Amazingly, it was invented quite independently by two different people at roughly the same time! John Nash and Piet Hein.

John Nash: Find out about his extraordinary life, by watching the feature film 'A Beautiful Mind.'



Piet Hein devised games, including Hex, TacTix and the Soma cube.



### Play the Game

You need some counters of two different colours and a Hex board (on the next page).

Each player has different coloured counters.

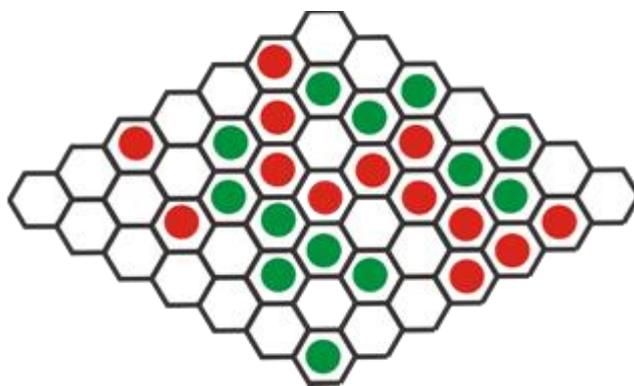
- Take turns to place a counter on any empty hexagon. The winner is first to make a complete line of counters from one side to the opposite side.
- The player who starts must not use the centre hexagon.
- They make their line from the bottom left side to the top right side. The other player makes their line from top right to bottom left.

### Example Game (Red wins)

#### Developing Strategy

Play the games lots of times.

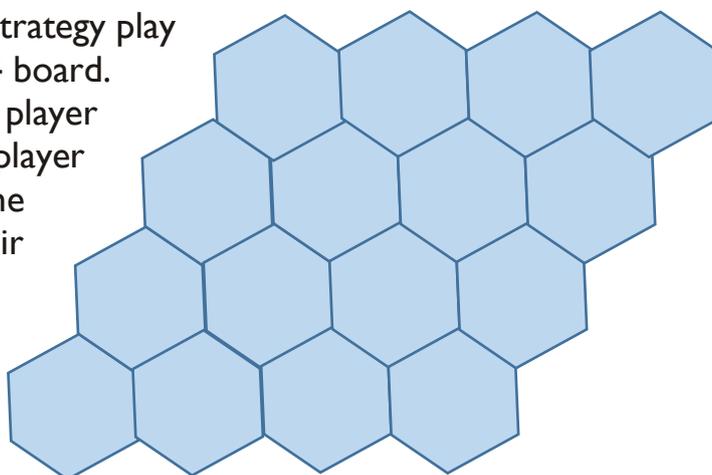
Develop a system to improve your chances of winning.



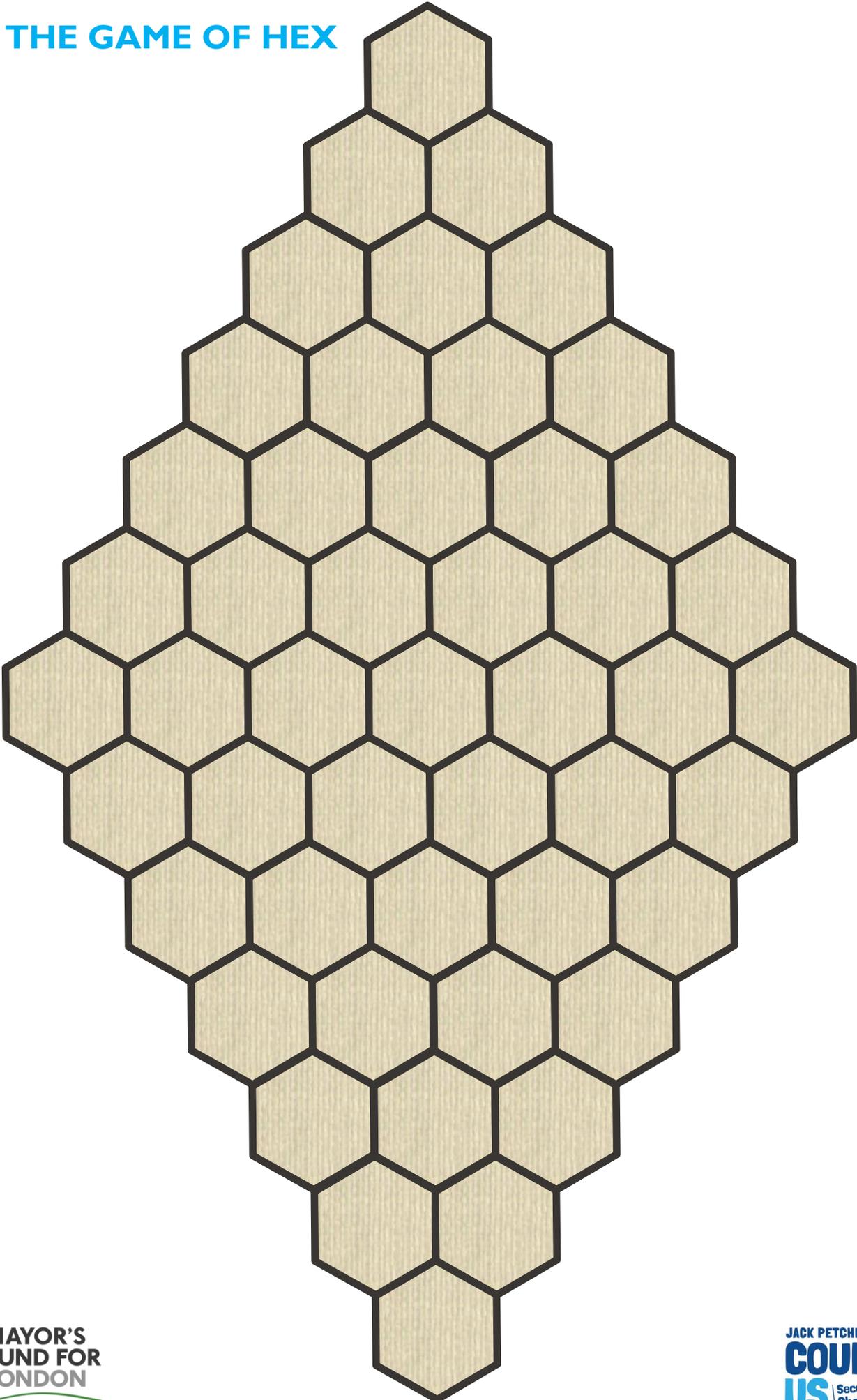
#### Tournament Game

When you have developed a winning strategy play the tournament game. Play on this 4x4 board.

Play with one additional rule: The first player can use the centre hexagon, BUT the player to move second can choose to steal the first player's move i.e. swap one of their counters for the one placed. The first player then plays their turn again.



# THE GAME OF HEX



## Geometric Problems

- Angles (Angle Relationships in lines and polygons)
- Pythagoras (Using Pythagoras' Theorem)
- Mensuration (Finding the area and perimeter of triangles, quadrilaterals and circles)

### Getting Prepared

1. Practice your geometric problem solving.
  - a. <https://www.bbc.com/bitesize/guides/zrck7ty/revision/1>
  - b. <https://www.bbc.com/bitesize/guides/z3g9q6f/revision/1>
  - c. <https://www.bbc.com/bitesize/guides/z2mtyrd/revision/1>
  - d. <https://www.bbc.com/bitesize/guides/zc9wxnb/revision/1>
2. Get confident using variables in geometry
  - a. Try this activity: <https://nrich.maths.org/perimeterexpressions>
  - b. Work through this: <https://www.ocr.org.uk/Images/222109-topic-check-in-6.01-algebraic-expressions.pdf>
  - c. Solve these:  
<http://www.somerset.k12.ky.us/userfiles/103/Word%20Problems%20Perimeter%20and%20Age.pdf>
3. Take the 9 level 0 cards in the GridLines: Geometry game pack (look for a 0 in the blue circle on the card). Work with a partner. Take turns to find a set of numbers that fits the situation. Find as many sets as you can. Now find level 1 cards and repeat. Now, level 2. These are quite hard! Finally, find level 3 cards and repeat. These are very hard – you would be best working together.
4. Now play a game of GridLines: Geometry. Read the rules on the next two pages, so you are clear about how it works.

# Gridlines Geometry Rules

GridLines: Geometry is a card game played with two 56 card decks consisting of:

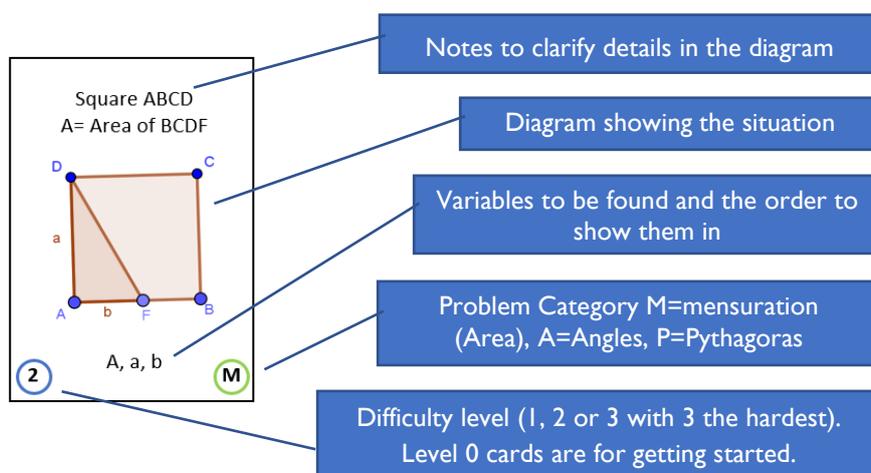
- 6 Rules cards (to remind you of the rules in play).
- 3 'I give up' cards (to use when you cannot find solutions to any problem).
- 44 Number cards.
- 9 Level 0 problems cards for getting started practice.
- 50 Geometric Problem cards.

The aim of the game is to find solutions to situations shown on **Problem Cards**, using numbers made from **Number Cards**. There are many possible solutions to each card. You must find numbers that can fit all the variables in the given situation.

## Playing the Game Summary (also see the notes section below).

1. Take the 3 'I Give Up' cards to use later.
2. Shuffle Number and Problem Card packs and place them separately face down.
3. Place top 10 Number Cards in 2 rows of 5 face up and 3 problem cards face up.
4. Solve Problem Cards using the Number Cards on the table. (Look at the *Example Solution* on the next page).
5. Take, keep and replace correctly solved Problem Cards. Return and replace used Number Cards.
6. Play an 'I Give Up' card at any time to replace any or all the number and/or Problem Cards.
7. Keep solving until the agreed time is up. Score 1 for each solved card.

## The Problem Cards

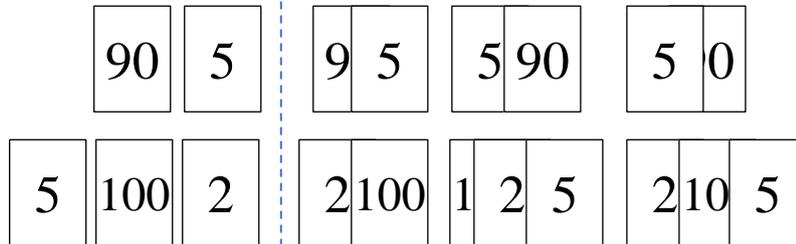


## The Number Cards

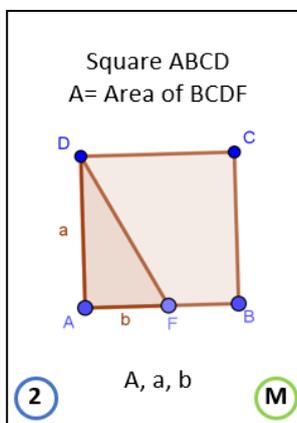
Number Cards can be organised into groups or played individually. Put the cards on top of one another to make a new number. When placed on the table, **ONLY** the required number can be visible.

Examples

Use:



## Example Solution



I choose  $a=10$  and  $b=4$  because I can make 4 and 10 from the number cards.

- The area of the triangle AFD is  $\frac{1}{2} \times 4 \times 10 = 20$
- The area of the square ABCD is  $10^2 = 100$
- So, the area of BCDF is  $100 - 20 = 80$

If I can make 80 from the number cards, then I can show my solution.

(If not, choose different values for a and b and try again).

I show A, a, b using the number cards for 80, 10, 4

You must use separate number cards for all the numbers.

A judge or opposing team should check that the solution is correct.

## Notes

1. In competitive play, you must explain, step-by-step to the judge how your variables fit the situation. You can show important calculations you have made in your notebook if this helps.
2. When you play one 'I give up!' card you can swap as many cards as you like. Even all of the problem **and** number cards or some number and some problem cards. Replace the cards taken to return to ten number cards and three problem cards face up on the table. Now resume play.
3. Experienced players may wish to count the total number of points from the cards solved. (These are shown in the blue circle on each card as 1, 2 or 3, where 3 is the hardest).

## ROUND 2: WHOLE NUMBER, FRACTIONS & DECIMALS, ALGEBRA & EXPONENTS

Ask an adult to do a calculation in their head and they'll run away! Everyone is scared of mental arithmetic. This round is designed to make sure YOU are not. All it needs is practice, practice, practice. (And a fun game to practice with ...)

The 24® Game is a card game. Each card has 4 numbers on it. You have to combine the numbers using +, -, × or ÷ in any way you can to make an answer of 24. You MUST use all four numbers once and once only!

See if you can do it with these numbers:

**4      5      8      4**

Hints

- Try to find key number bonds:  $6 \times 4$ ,  $8 \times 3$ ,  $16 + 8$  ...
- Try pairing the numbers up to make the parts you need.
- Try finding numbers to make 1 (to multiply and make no difference).
- Keep it all in your head!

Now try these:

**1      5      2      6**

**2      3      6      2**

**2      6      2      8**

**1      5      3      9**

**2      5      4      6**

Don't forget ... we won't tell you the answers, so don't tell anyone else.

Use the next four pages to practise, then use the 24® Game cards.

## Different 24® Game Cards

There are different types of 24® Game cards with different number types. You will have some cards of the different types to practice, they are:

- Single Digits: 4 single digit numbers.
- Double Digits: one or more of the numbers will be a two digit number.
- Fractions and Decimals: one or more of the numbers will be expressed as a fraction or a decimal.
- Integers: one or more of the numbers will be negative.
- Algebra and Indices:
  - Algebra cards have one or more of the spaces replaced with an algebraic expression with variable  $x$  and/or  $y$ . You choose any number(s) for the variable(s) (not zero) and use the value of the expression(s) to complete the 24.
  - Indices cards have four index expressions (always  $x^2, x^3, \sqrt{x}, \sqrt[3]{x}$ ). You choose one of these and apply one of the numbers on the card to make a new number, then use this and the others to complete the 24.

You should practice using each of the sets separately, then make sure you can still solve puzzles when the different types come up randomly. (Shuffle different types together to practice, but make sure to separate the packs to put them away!)

# TORTURE SQUARES

Use these torture squares to practice your fractions calculations.

Do them at different times. Write answers as either fractions or decimals.

You must allow exactly 10 minutes to fill in all the gaps.

In each square use only the operation shown: +, -, × or ÷

+	<b>3</b>	$\frac{1}{2}$		<b>4</b>	$\frac{3}{4}$
$\frac{1}{3}$					
		$\frac{4}{5}$			
					$1\frac{1}{4}$
<b>5</b>			$6\frac{1}{2}$		
$\frac{2}{3}$					

×	<b>8</b>	<b>0.1</b>	<b>2</b>	$\frac{1}{4}$	
$\frac{1}{2}$					
<b>0.3</b>					
			<b>0</b>		
<b>0.8</b>					<b>3.2</b>
$\frac{1}{3}$					

first number					
-	<b>4</b>		$\frac{1}{4}$	<b>7</b>	<b>0.3</b>
<b>1</b>					
<b>1.2</b>					
			$2\frac{3}{4}$		
$\frac{1}{5}$		<b>2.2</b>			
<b>2</b>					

first number					
÷	<b>6</b>			$\frac{2}{3}$	<b>0.5</b>
<b>2</b>		<b>0</b>			
<b>0.2</b>					
$\frac{1}{4}$			<b>4</b>		
<b>5</b>					
$\frac{1}{3}$					

Make up more Torture Squares like these to test your friends.

## FIND 24: THE BOARD GAME

1. You will need sets of counters of two different colours; one for each player.
2. Take turns to find 24 using any four numbers on the board.

**For example:** Use  $4, \frac{1}{2}, 10, 1.2$  make  $4 \times \frac{1}{2} = 2$  and  $10 \times 1.2 = 12$

then  $2 \times 12 = 24$

Use a timer to give a maximum of one minute.

3. If you succeed, place 4 counters on the number you found.
4. If you fail, your opponent takes a turn.
5. Numbers cannot be used more than once.
6. When neither player can make 24 in two consecutive rounds, play ends and the winner is the player who has placed the most counters.

### Alternative Rules :

1. Both players look for sets to make 24 at the same time.
2. If you find a set tap the table and play stops. Place your counters.
3. Score 1 if all 4 numbers are whole numbers, add one for each fraction or decimal you used.
4. Play until both players agree they cannot find any more sets OR agree a time limit in advance.
5. For an even harder game, the set of four numbers must be next to each other on the board (horizontally, vertically or diagonally).

### Exponents Version :

You **must** substitute *one* of your numbers (**but NOT the number 1**) into one of the following expressions:  $x^2, x^3, \sqrt{x}, \sqrt[3]{x}$

### Algebra Version :

You must substitute one or two of your numbers into one of these expressions before making 24 (when an expression has been used it cannot be used again):

$x^2 + 1$	$x(x + 1)$	$x^2 + y$	$\frac{x^2}{y}$
$2x(1 - x)$	$\frac{x^2}{y^2}$	$x^2 + 2y^2$	$y(x^2 - 3)$
$x(x^2 + 2y)$	$\frac{x + 2y}{x}$	$x(3 - y^2)$	$\frac{3x - y}{2y}$

## FIND 24 BOARD 1 (BEGINNER)

4	$\frac{1}{4}$	6	$\frac{1}{6}$	0.1	1
12	$\frac{1}{8}$	3	9	$\frac{1}{2}$	0.75
2	4	0.5	2	3	20
8	$\frac{2}{3}$	1	3	$\frac{1}{3}$	0.5
$\frac{1}{4}$	9	$\frac{3}{4}$	0.5	2	1
6	$\frac{1}{2}$	4	10	0.25	6

## FIND 24 BOARD 2 (INTERMEDIATE)

6	$\frac{1}{4}$	3	1	$\frac{1}{3}$	8
$\frac{1}{4}$	-2	0.5	$\frac{1}{2}$	9	$\frac{1}{2}$
-12	$\frac{1}{8}$	0.1	4	0.5	20
3	0.2	$\frac{2}{3}$	-3	$\frac{1}{6}$	10
$\frac{1}{12}$	-9	-4	$\frac{5}{6}$	2	$\frac{3}{4}$
1.5	$\frac{2}{3}$	4	6	0.5	-6

## FIND 24 BOARD 3 (EXPERT)

$-7$	$-\frac{1}{2}$	$3$	$0.8$	$\frac{3}{5}$	$1.5$
$\frac{1}{3}$	$-2$	$0.5$	$\frac{1}{4}$	$-8$	$\frac{1}{12}$
$2\frac{1}{4}$	$\frac{2}{3}$	$-1.2$	$1\frac{1}{3}$	$2.5$	$11$
$4$	$-0.4$	$\frac{3}{8}$	$2$	$\frac{1}{6}$	$-1\frac{3}{4}$
$\frac{1}{2}$	$9$	$1.3$	$-\frac{2}{3}$	$1.4$	$\frac{5}{12}$
$0.3$	$\frac{5}{6}$	$-3$	$1\frac{1}{2}$	$0.8$	$5$

## ROUND 3: I KNOW LONDON ALGEBRA PROBLEM SOLVING CHALLENGE

Algebra is at the heart of all mathematics. It is the language that mathematicians use. You must speak it fluently! Also, you live in London, one of the world's greatest cities and you need to know it well. In this round, you need to use your fast-paced skill in algebra to decode messages to solve a problem about the city.

1. You will need to use information you are given about famous institutions in London. (The Mayor of London/City Hall, St Pancras Railway Station, Imperial College London, The Science Museum, Lloyd's of London, St Thomas' Hospital). Look them up online to get an idea of what they are and what they do.
2. You will need to know how to solve a Caesar or Shift cipher. The following page will help you. Do the practise example on that page.
3. You will need to practice your algebra. Two pages forward you will find a list with all the algebra problems you will need to know. Use this list to decide what to practice. See below for puzzles to help you practice.

### On the web.

Go for a walk in London looking at maths: [www.themathszone.com/?p=641](http://www.themathszone.com/?p=641)

Read about London at: <http://en.wikipedia.org/wiki/London>

Explore code breaking at: [http://www.simonsingh.net/The\\_Black\\_Chamber](http://www.simonsingh.net/The_Black_Chamber)

### Practice algebra by making and solving Tarsia puzzles.

First you will need to download the free Tarsia software at:

<http://www.mmlsoft.com/index.php/products/tarsia>

Then download the Algebra set of puzzles (scroll down to find them) at:

<http://www.mrbartonmaths.com/jigsaw.htm>

When the software is installed, choose one of the puzzles. Look at the examples on the algebra page (later in this booklet) to guide your choice. Open the file. Make sure the 'output' tab is selected. Print out the sheets. Cut them out. Put them together to make a large hexagon so that edges match with question and answer. ONLY when you have finished click the 'solution' tab.

We recommend you work with a partner to solve these puzzles.

## THE CAESAR CIPHER

Julius Caesar and his generals used shift codes to move the alphabet along by a certain number of places. The number of places is called the **offset**.

**Example 1.** All the letters have been moved up two in this code. The offset is 2. A in the message is C in code, B is D in code, C is E and all the way to Z which B in code. This is a message written in code:

VJKU KU CP GCUA QPG VQ IGV AQW UVCTVGF

To decipher the code we have to move the letters two places back in the alphabet.

So G in code is E in the message, C in code is A, U is S and A is Y so the word GCUA is EASY. Find out what the rest of the code says.

### EXAMPLE 2.

This message has been encoded with an offset which is the solution to the equation:  $13x - 7 = 4x + 83$ . Work out the offset and decode the message.

IYE KXN IYEB DOKW KBO BOKNI DY WOOD DRO MYEXD YX EC MRKVVOXQO

### EXAMPLE 3.

This coded word is made from the solutions to a set of equations. Find the solution and make the coded message using A=1, B=2, C=3 etc. The final equation tells you the offset. Now decode the word and look them up on the internet adding 'maths' to your search.

- $5x - 1 = 19$
- $2x + 1 = x + 17$
- $3x - 2 = x + 36$
- $2 - x = 3x - 46$
- $7x - 13 = 6x + 13$
- **Offset:**  $x^2 + 49 = 14x$

When you have completed the practice examples on this page, make codes for your team and challenge them to break them.

Use: [http://www.simonsingh.net/The\\_Black\\_Chamber/caesar.html](http://www.simonsingh.net/The_Black_Chamber/caesar.html) to help you.

# ALGEBRA

This page is designed to show you the hardest possible things you will need to solve in the tournament! Don't worry if you cannot do them yet or even don't know what they mean. You should use this as a guide to help you practice.

You must be able to solve all equations of these types:

Linear Equations:	$3x + 7 = 5x - 11$ ; $\frac{2}{x} + 7 = -2$
Quadratic Equations (factorisable)	$x^2 - 2x - 12 = 6 + x$
Exponential Equations	$3^x - 11 = 70$

You must be able to rearrange and substitute into formulae:

- Find the coefficient of  $x$  when  $R$  is the subject of:  $3x + 2R = 5x - R$
- Find the value of  $a$  when  $A = 24$ ,  $b = 7$  and  $h = 3$  in  $A = \frac{1}{2}h(a + b)$
- The coefficient of  $x$  when we multiply out  $(2x + 3)(5x - 2)$

You must be able to identify values in sequences:

- Find  $u_7$  where  $u_n = 2^n - 3$
- If  $u_1 = 7$  and  $u_{n+1} = 3u_n + 1$  what is  $u_4$
- Find the 8<sup>th</sup> term in the sequence: 3, 7, 12, 19, ...
- Find the fourth term in the Fibonacci sequence: 2, ..., ..., ..., 28

You must be able to find the gradient and  $y$  –intercept of lines specified in other forms:

- Find the gradient and  $y$  –intercept of the line  $2 - 3x + 7y = x - 2y + 13$

## Practice Your Algebra:

1. Using the Algebra Practice activities on the next pages.
2. By making and solving Tarsia puzzles.
3. Using GeoGebra.
  - a. Download the software at [www.geogebra.org](http://www.geogebra.org)
  - b. In the view options choose CAS
  - c. For example, type this:  $\text{solve}(x^2+5x+6=0)$  and press return
  - d. Try different equations. Experiment. Explore!

## ALGEBRA PRACTICE (Easiest, but still quite tough!)

Print a copy of this page. Cut out the pieces. Match the question to the answer.

Question	Question	Answer	Answer
$5x + 3 = 43$	$\frac{1}{x} = \frac{1}{13}$	34	20
Find the sixth term in the Fibonacci sequence: 1, 1, 2, ....., .....	The coefficient of $x$ in this expression: $5x + 2d - 2x = 1$	13	10
$\frac{x}{3} = 7$	Find the 6 <sup>th</sup> term in the sequence: 2, 5, 8, 13, ...	5, 1	7
Find the value of $V$ when $I = 2$ and $R = 10$ in $V = IR$	$x^2 - 7x + 12 = 0$	8	9
If $u_n = 2n - 1$ what is $u_4$	$7x - 5 = 58$	11	8
The gradient and $y$ -intercept of the line $y = 2x + 3$	The coefficient of $y$ when simplified $3y + 2 = 1 - 7y$	21	3, 4
$(x - 5)(x - 1) = 0$	$4x - 22 = 2x$	9	3
$3^x = 9$	The coefficient of $x$ multiplied out $3x(2x + 3)$	2	2, 3

## ALGEBRA PRACTICE (Middle)

Print a copy of this page. Cut out the pieces. Match the question to the answer.

Question	Question	Answer	Answer
Find $u_3$ where $u_n = n^2 + 8$	$x^2 - 5x + 7 = 1$	7	4,5
$\frac{x}{3} + 1 = \frac{x}{2}$	Find the fifth term in the Fibonacci sequence: 1, 3, ..., ..., ...	4	34
The gradient and $y$ -intercept of the line $8x + 14 = 2y$	$\frac{20 + x}{x} = 5$	8	6
Find the 7 <sup>th</sup> term in the sequence: 2,3,5,8,.....	$2^x - 3 = 125$	17	12
$3x + 17 = 9x - 7$	The coefficient of $x$ when we multiply out $(3x + 2)(5x + 1)$	3	2,3
If $u_1 = 3$ and $u_{n+1} = u_n + 1$ what is $u_6$	Coefficient of $x$ when simplified $7x - 2b = 3c - 5x$	5	13
$x^2 - 5x + 10 = 2x$	$3x - 17 = 7 - 5x$	12	4,7
Find the value of $v$ when $u = 2$ , $a = 10$ and $t = 1$ in $u + at$ $v =$	The gradient and $y$ -intercept of the line $3y - 5 = 12x$	11	2,5

## ALGEBRA PRACTICE (Hardest)

Print a copy of this page. Cut out the pieces. Match the question to the answer.

Question	Question	Answer	Answer
The gradient and $y$ –intercept of the line $3 - 7x = 15 - x - 3y$	The coefficient of $x$ when we multiply out $(6x + 7)(5 - 3x)$	1	25
$x^2 - 3x + 7 = 2x + 1$	Find the third term in the Fibonacci sequence: ..., 3, ..., ..., 11	2,5	6
$-99 = 26 - 5^x$	$\frac{x}{3} - 2 = \frac{x}{5}$	2,4	9
Find the value of $t$ when $v = 20$ , $u = 4$ and $a = 8$ in $v = u + at$	Find $u_6$ where $u_n = 2n^2 - 65$	5	7,4
$8x - 4 = 5x + 11$	$x^2 - 8x + 28 = 3x$	20	2
If $u_1 = 2$ and $u_{n+1} = 4u_n - 3$ what is $u_3$	Coefficient of $M$ when $y$ is the subject $\frac{3y - 7M}{2} = 9M + y$	4	15
Find the 6 <sup>th</sup> term in the sequence: -15,-12,-7,0,...	$2^x + 11 = 75$	3,2	3
$3 + \frac{4}{x} = 7$	The gradient and $y$ –intercept of the line $-8x = 20 - 4y$	17	7